Low-rank SYK models for quantum simulations of holographic systems

Based on 2411.17802, 2512.13774 with [Baumgartner, Bandyopadhyay, Orsi, Sauerwein, Hauke, Brantut, Sonner] and some WiP

Pietro Pelliconi

Realising holographic systems

Many body systems with a holographic gravity duals have very interesting properties

- black hole physics probed by thermal states (more generally horizons),
- maximal Lyapunov exponent,
- late time RMT behavior.

General hope to understand new features of Quantum Gravity.

Realising holographic systems

Realising them is quite difficult:

- Some examples require *dense* disorder and all-to-all connectivity (*e.g.* SYK),
- Some require Matrix Quantum Mechanics and large-N (*e.g.* BFSS),
- Other more challenging examples (*e.g.* $\mathcal{N} = 4$ SYM).

This makes them interesting also for experimentalists.

The Sachdev-Ye-Kitaev model

We will be interested in the Sachdev-Ye-Kitaev (SYK) model, dual to two-dimensional models of gravity

$$H = (i)^{\frac{p}{2}} \sum_{j_1 < \cdots < j_p} J_{j_1 \ldots j_p} \, \psi_{j_1} \ldots \psi_{j_p}$$ Majorana SYK

$$H = \sum_{\substack{j_1 < \cdots < j_p \\ k_1 < \cdots < k_p}} J_{j_1 \ldots j_p ; k_1 \ldots k_p} \, c^\dagger_{j_1} \ldots c^\dagger_{j_p} c_{k_1} \ldots c_{k_p}$$ Complex SYK

[Sachdev, Ye 1993] [Kitaev 2015] [Sachdev 2015]

Some analogue proposals

Solid state platforms

- Majorana zero-modes at the edge between 3D TI and ordinary superconductor [Pikulin, Franz 2017]
- Quantum dot coupled to topological superconducting wires (hosting Majorana zero modes) [Chew, Essin, Alicea 2017]
- Graphene flakes with irregular boundaries [Chen et al. 2018]

Some analogue proposals

Analog quantum simulators

- Optical kagome lattices with strong disorder [Wei, Sedrakyan 2021]
- Cold atoms in optical cavities:
  - Lattices of atoms and molecules [Danshita, Hanada, Tezuka 2016]
  - Multimode cavity-QED (cQED) [Uhrich et al. 2023]
  - Single mode cQED [Baumgartner, PP et al. 2024]

SYK in cQED

HologrAPh consortium

EPFL

Group Prof. J.P. Brantut

ETH Zurich

Group Prof. T. Esslinger

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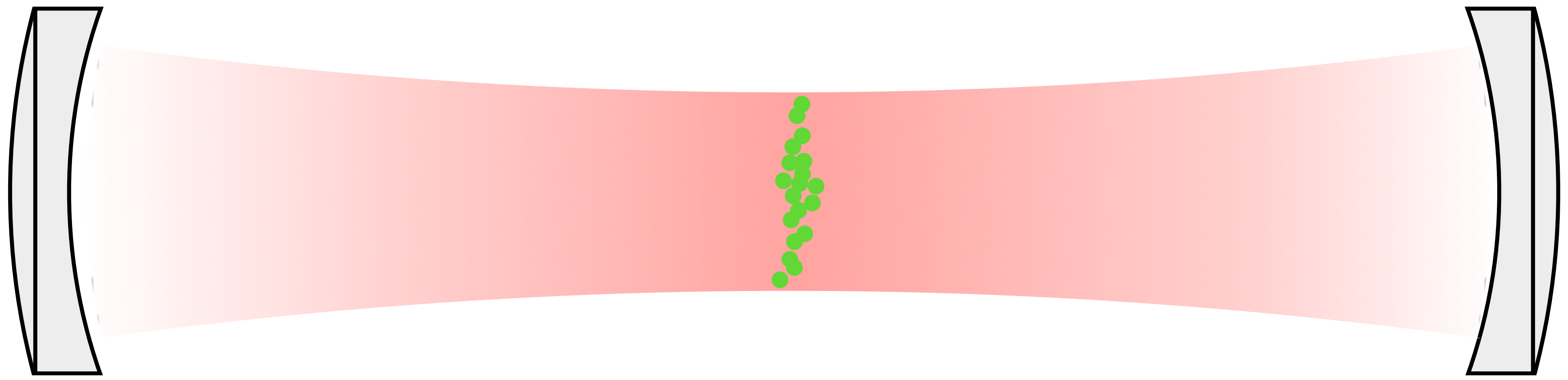
Group Prof. J. Sonner

University of Trento

Group Prof. P. Hauke

Cavity QED platforms

[Uhrich et al. 2023]
[Baumgartner, PP et al. 2024]



$$H = \boxed{H_{\mathrm{kt}}} + \boxed{H_{\mathrm{c}}} + \boxed{H_{\mathrm{a}}} + \boxed{H_{\mathrm{ac}}} + \boxed{H_{\mathrm{ad}}}$$

$$H_{\mathrm{kt}} = \sum_{\mathrm{s}\in\{\mathrm{e},\mathrm{g}\}} \int \mathrm{d}^2 r\, \psi_{\mathrm{s}}^{\dagger}(r) \left(\frac{p^2}{2m_{\mathrm{at}}} + V_{\mathrm{t}}(r) \right) \psi_{\mathrm{s}}(r)$$

$$H_{\mathrm{c}} = \omega_{\mathrm{c}}\, a^{\dagger} a$$

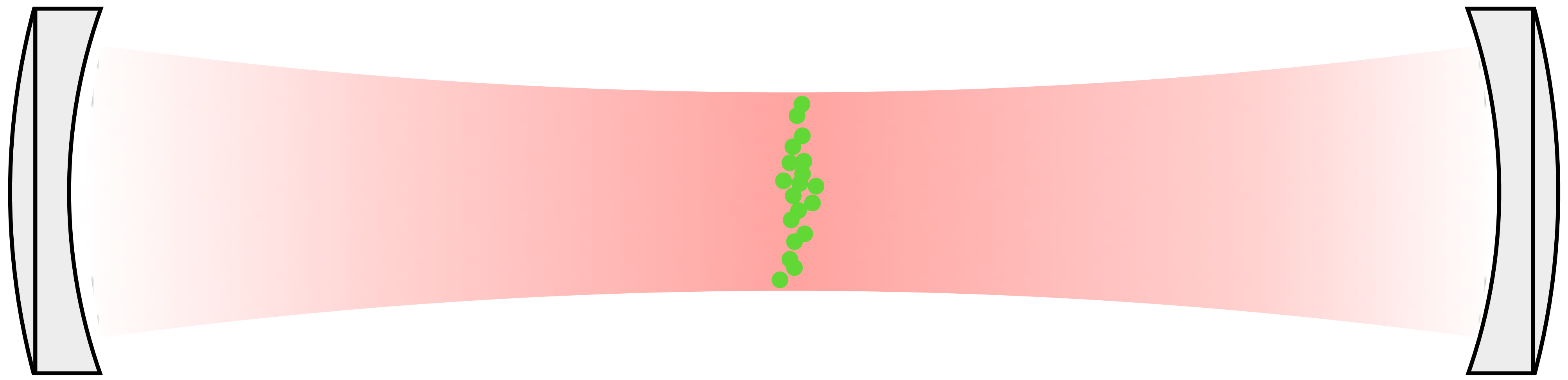
$$\psi_{\mathrm{g}}(r) = \sum_i \phi_i(r)\, c_i$$

$$H_{\mathrm{a}} = \int \mathrm{d}^2 r\, \omega_{\mathrm{a}}(r)\, \psi_{\mathrm{e}}^{\dagger}(r)\, \psi_{\mathrm{e}}(r)$$

Speckle pattern

Cavity QED platforms

[Uhrich et al. 2023]
[Baumgartner, PP et al. 2024]



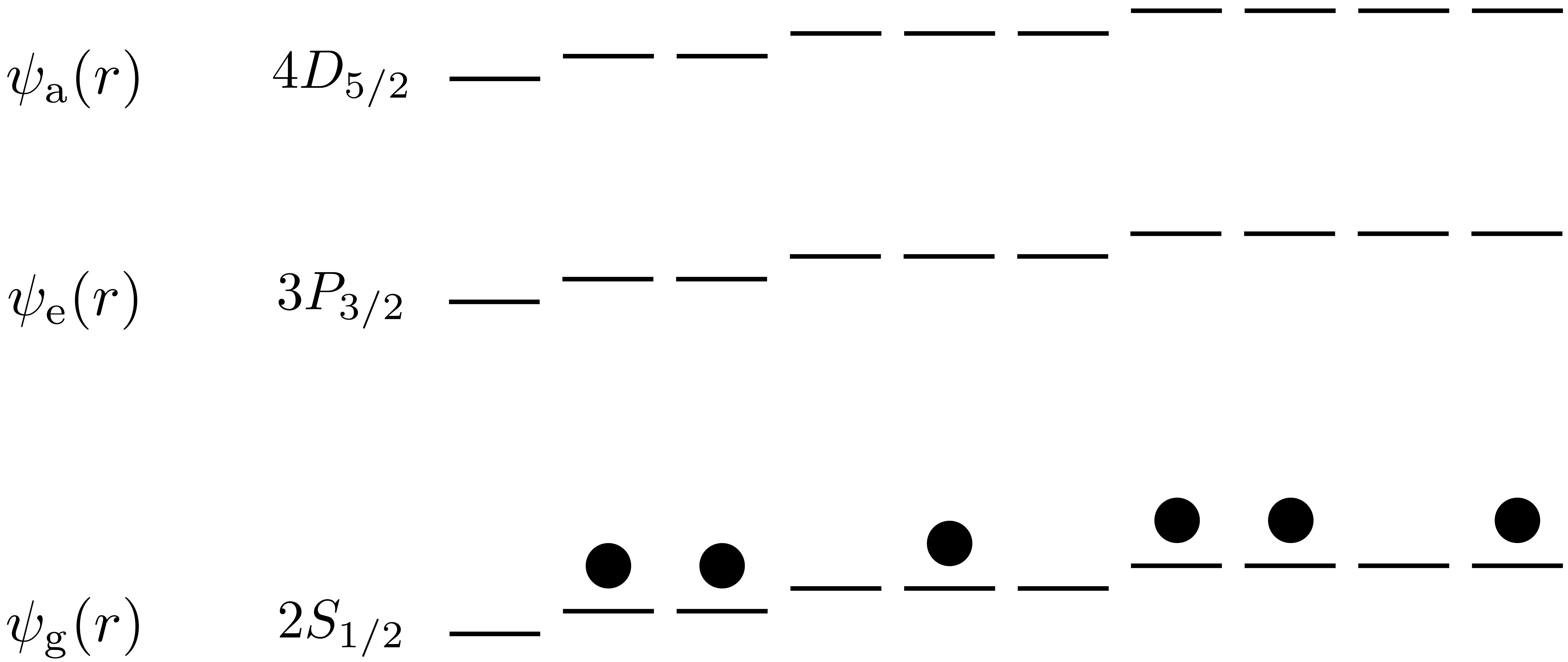
$$H = \boxed{H_{\rm kt}} + \boxed{H_{\rm c}} + \boxed{H_{\rm a}} + \boxed{H_{\rm ac}} + \boxed{H_{\rm ad}}$$

$$\boxed{H_{\rm ac} = \frac{\Omega_{\rm c}}{2} \int \mathrm{d}^2 r \, \left(g_{\rm c}(r) \psi_{\rm e}^\dagger(r) \psi_{\rm g}(r) a + \mathrm{h.c.} \right)}$$

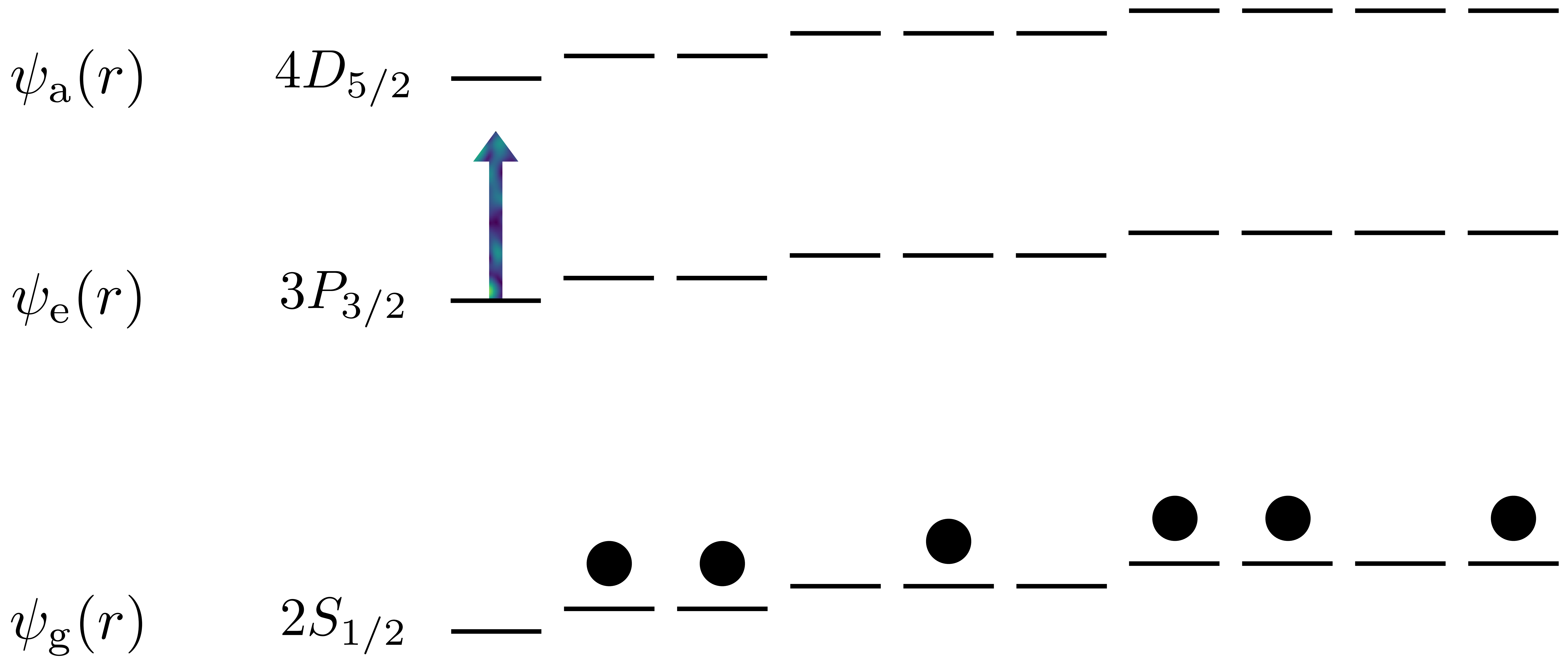
$$\boxed{H_{\rm ad} = \Omega_{\rm d} \int \mathrm{d}^2 r \, \left(g_{\rm d}(r) \, e^{-i\omega_{\rm d} t} \psi_{\rm e}^\dagger(r) \psi_{\rm g}(r) + \mathrm{h.c.} \right)}$$

Going to rotating frame: measuring frequencies wrt ω_d

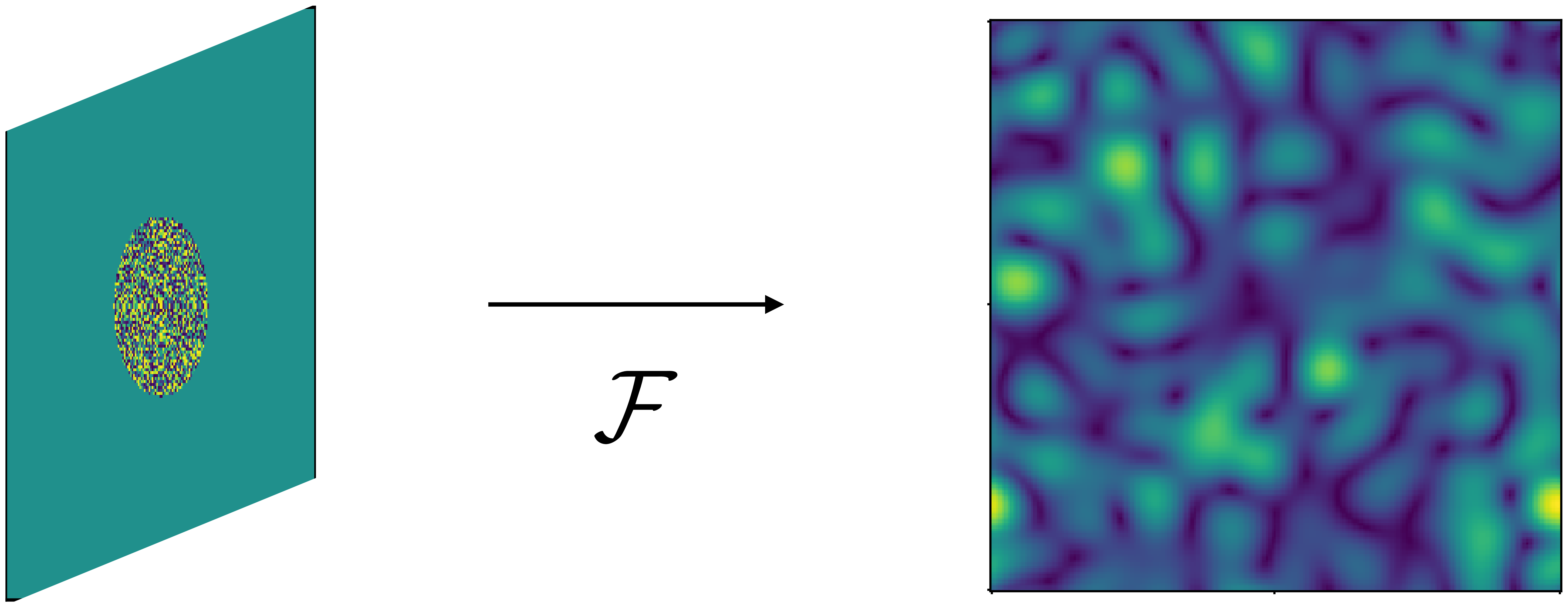
Energy levels



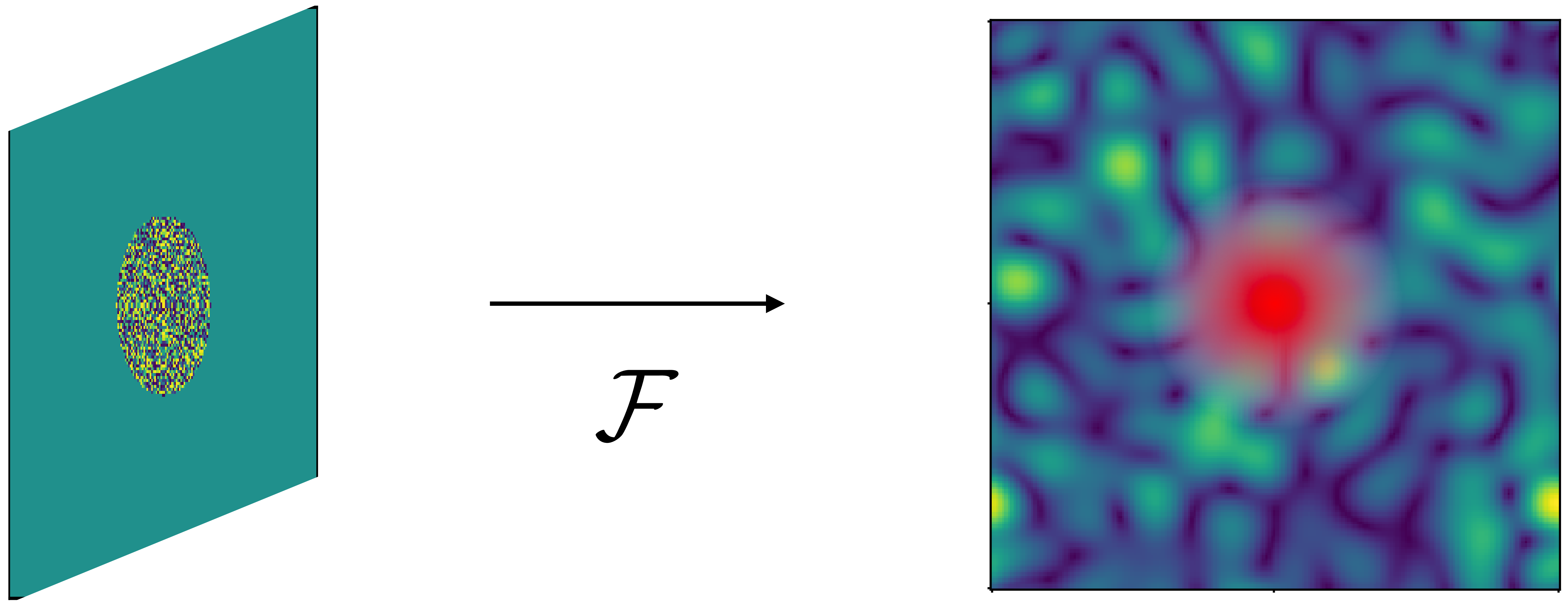
Energy levels



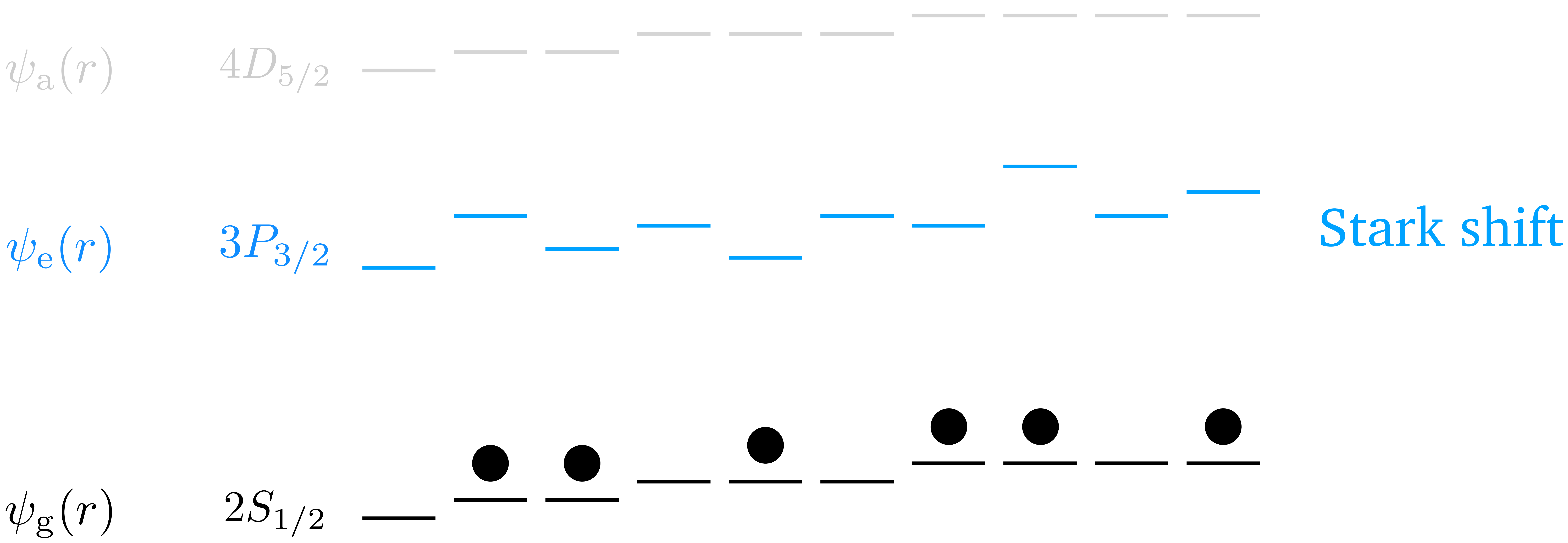
Speckle patterns



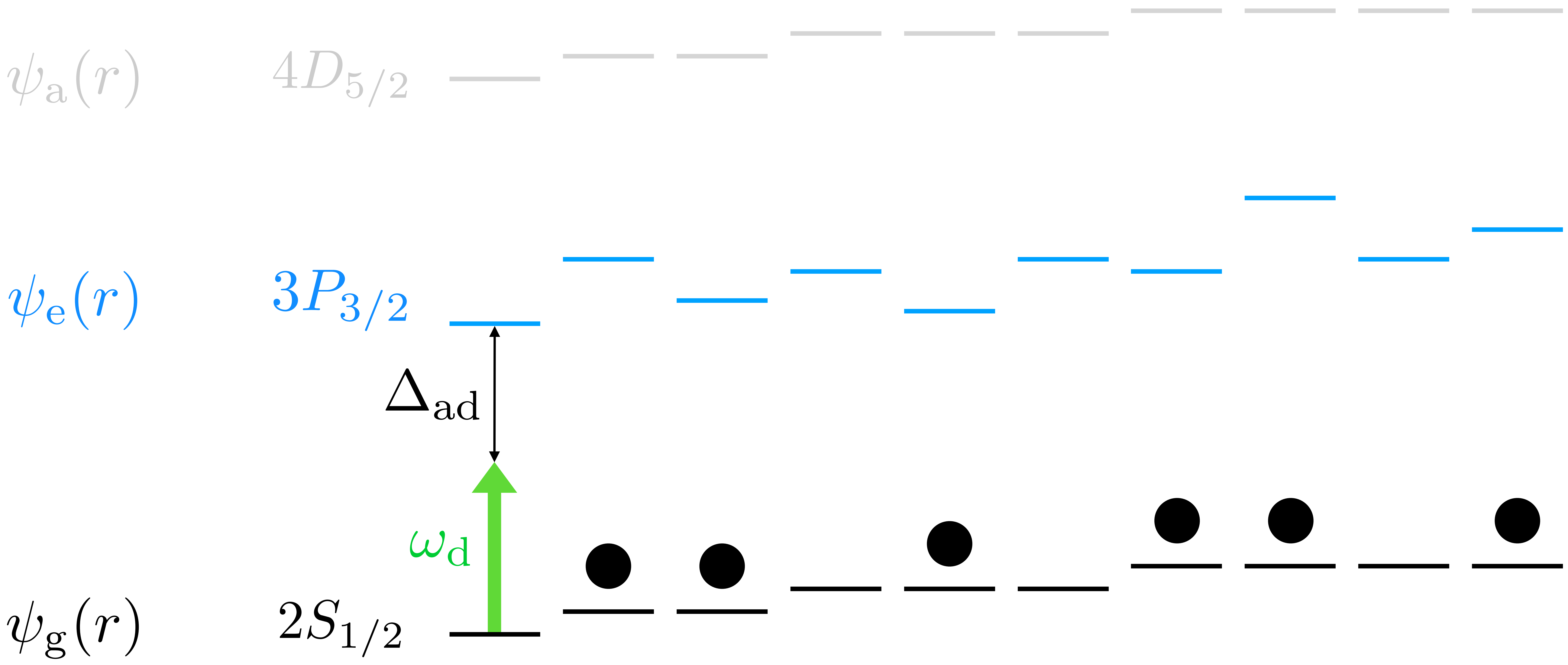
Speckle patterns



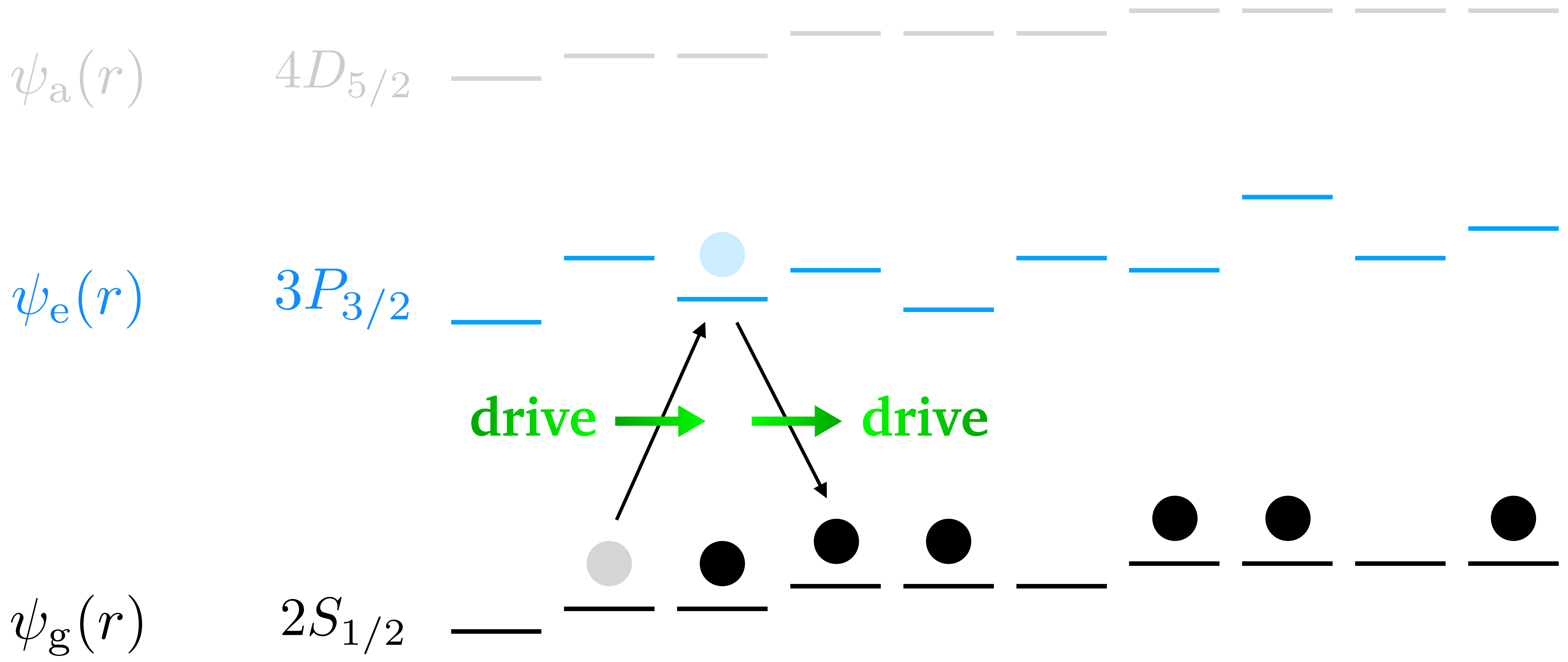
Energy levels



Energy levels

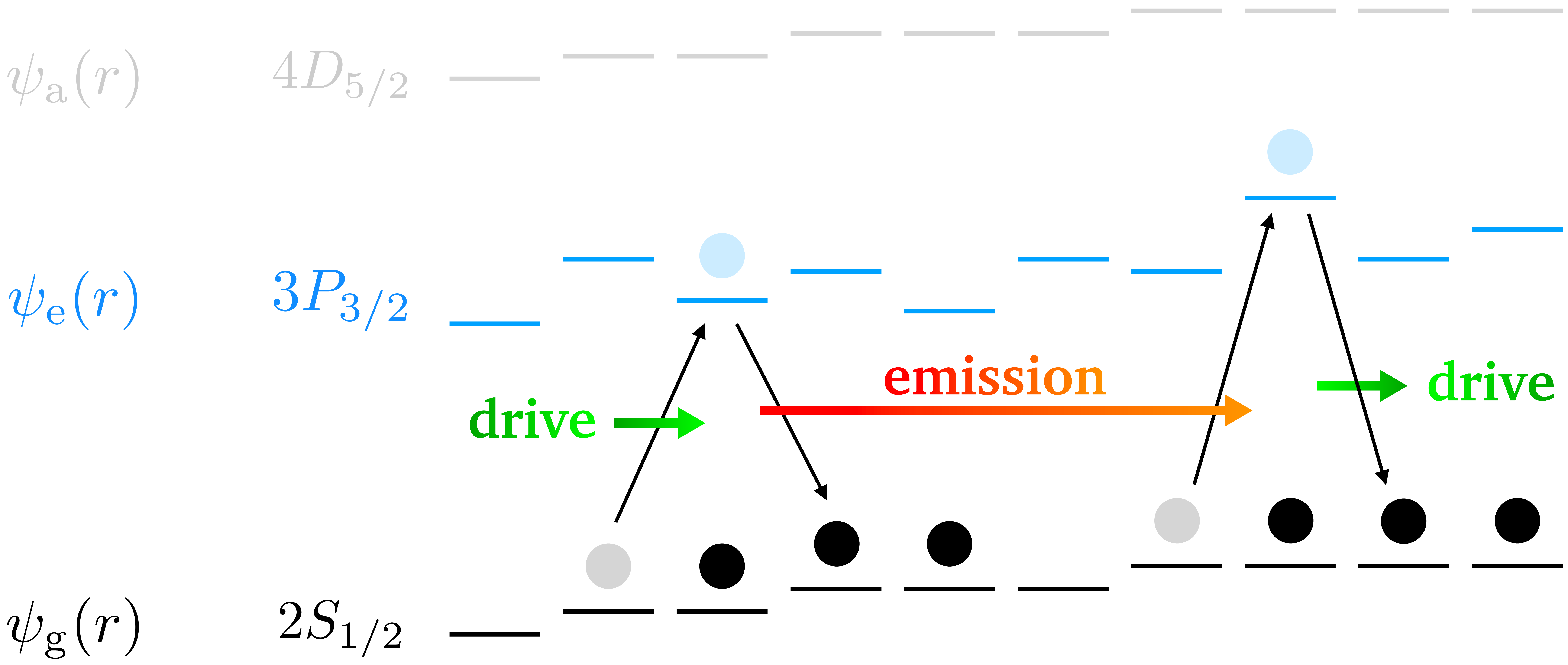


Two body interaction



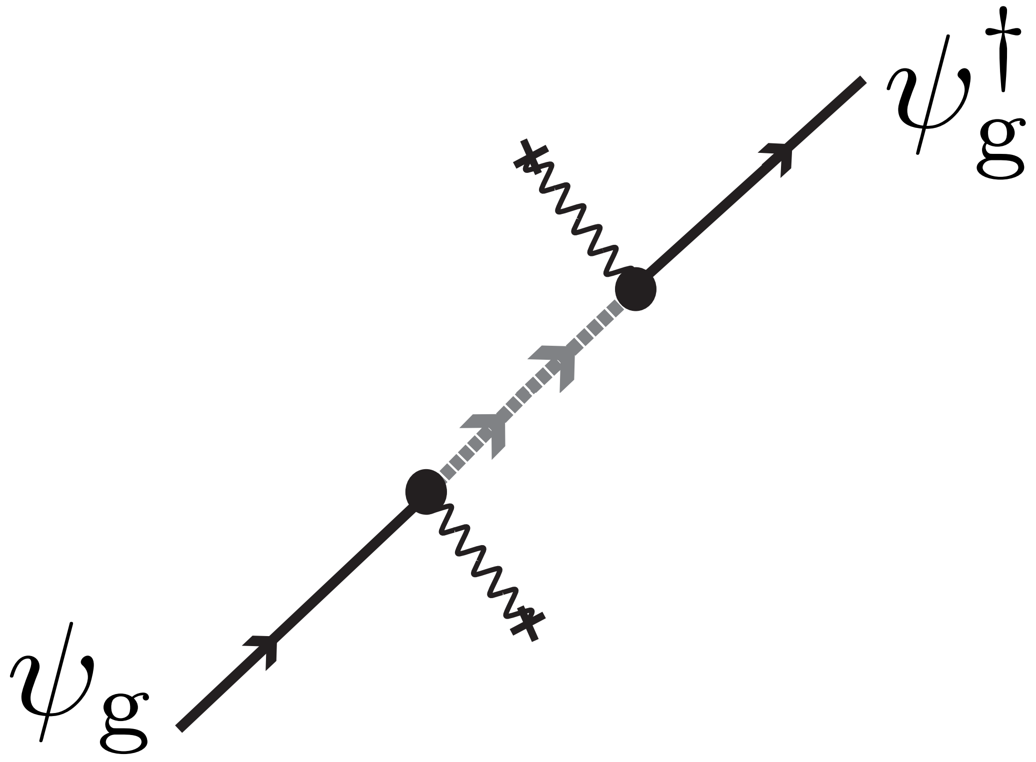
$$\frac{\Omega_d^2}{\Delta_{ad}} \int d^2r \frac{\Delta_{ad}}{\Delta_{ad}(r)} \psi_g^\dagger(r)\psi_g(r)$$

Two body interaction

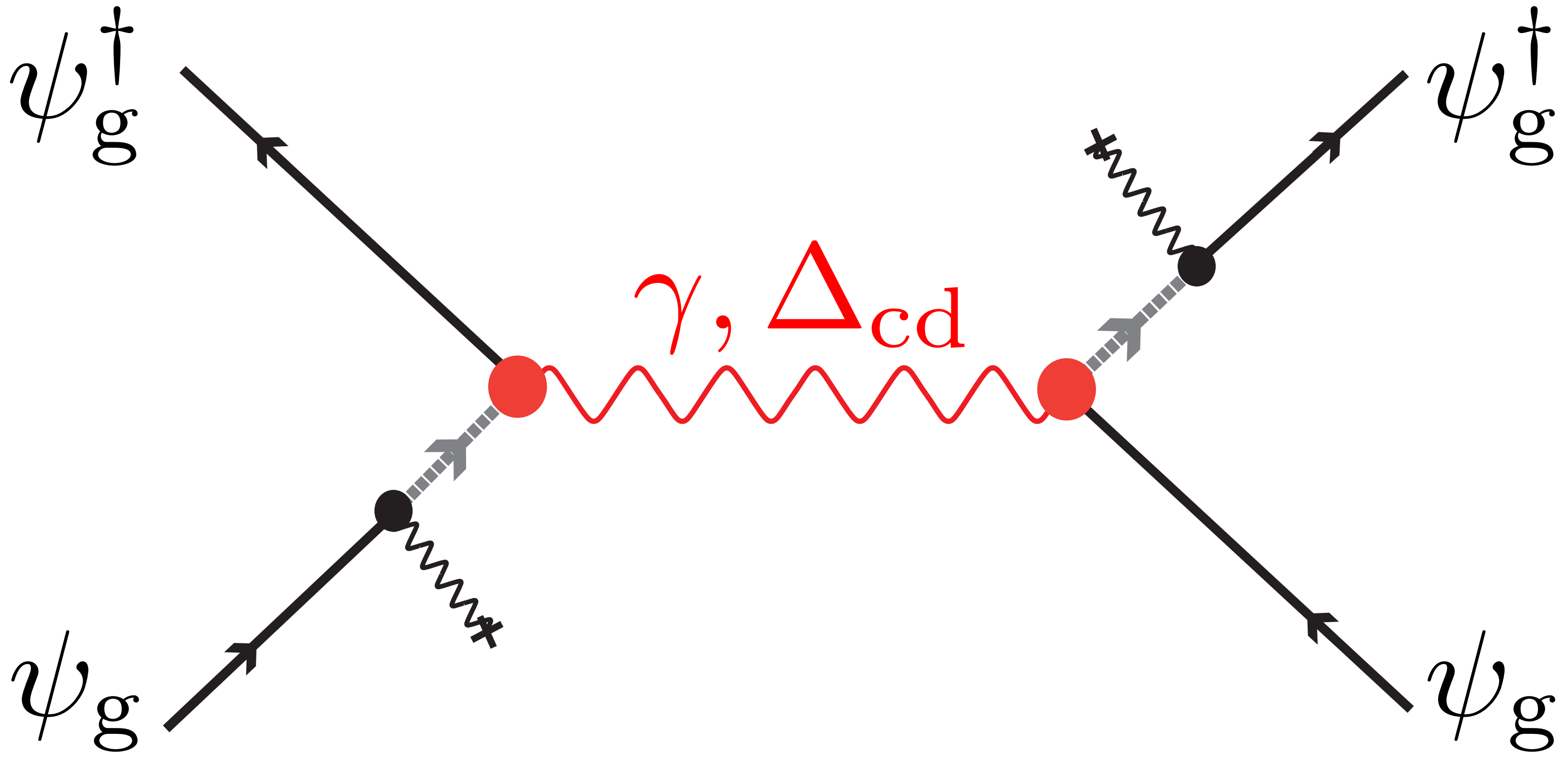


$$\frac{\Omega_d^2\Omega_c^2}{4\Delta_{cd}\Delta_{ad}^2}\int d^2r\, d^2r'\,\frac{\Delta_{ad}^2}{\Delta_{ad}(r)\Delta_{ad}(r')}\,\psi_g^\dagger(r)\psi_g(r)\psi_g^\dagger(r')\psi_g(r')$$

Diagrams

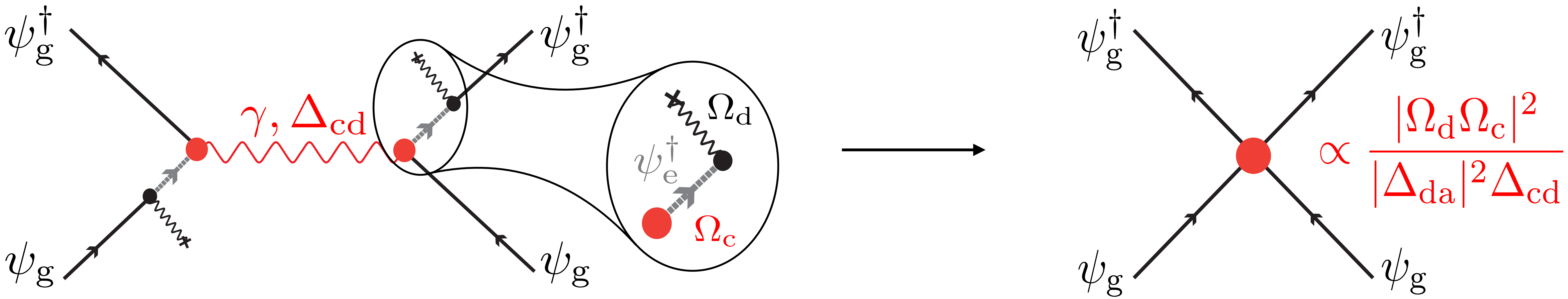


$$\frac{\Omega_{\mathrm{d}}^2}{\Delta_{\mathrm{ad}}} \int \mathrm{d}^2 r \, \frac{\Delta_{\mathrm{ad}}}{\Delta_{\mathrm{ad}}(r)} \, \psi_{\mathrm{g}}^{\dagger}(r) \psi_{\mathrm{g}}(r)$$



$$\frac{\Omega_{\mathrm{d}}^2 \Omega_{\mathrm{c}}^2}{4 \Delta_{\mathrm{cd}} \Delta_{\mathrm{ad}}^2} \int \mathrm{d}^2 r \, \mathrm{d}^2 r' \, \frac{\Delta_{\mathrm{ad}}^2}{\Delta_{\mathrm{ad}}(r) \Delta_{\mathrm{ad}}(r')} \, \psi_{\mathrm{g}}^{\dagger}(r) \psi_{\mathrm{g}}(r) \psi_{\mathrm{g}}^{\dagger}(r') \psi_{\mathrm{g}}(r')$$

Diagrams



$\psi_g^\dagger$
ψ_g
γ, Δ_{cd}
$\psi_g^\dagger$
ψ_g
Ω_d
$\psi_e^\dagger$
Ω_c
$\psi_g^\dagger$
$\psi_g^\dagger$
ψ_g
ψ_g
$\propto \frac{|\Omega_d \Omega_c|^2}{|\Delta_{da}|^2 \Delta_{cd}}$

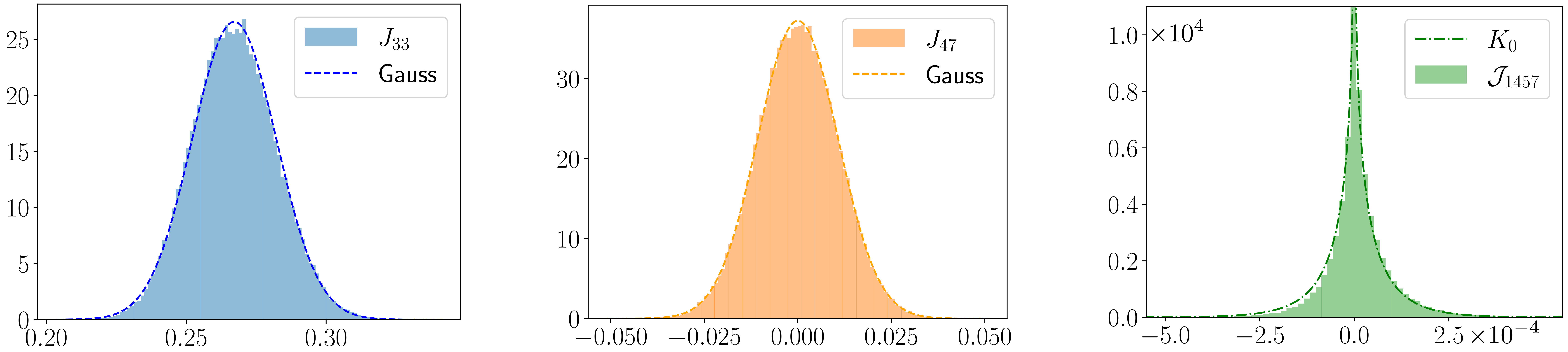
Experimental couplings

$$J_{ij} = \frac{1}{2} \int \mathrm{d}^2 r \, \frac{\Delta_{\mathrm{ad}}}{\Delta_{\mathrm{ad}} + S(r)} \, \phi_i^*(r) \phi_j(r)$$

$$H_{\mathrm{eff}} = \frac{\Omega_{\mathrm{d}}^2 \Omega_{\mathrm{c}}^2}{\Delta_{\mathrm{cd}} \Delta_{\mathrm{ad}}^2} \sum_{i_1, i_2; k_1, k_2} J_{i_1 k_1} J_{i_2 k_2} c_{i_1}^\dagger c_{k_1} c_{i_2}^\dagger c_{k_2}$$

Trotterization: $$H = \sum_{n=1}^{R} H_{\mathrm{eff}}^n = \mathcal{E} \sum_{i_1 i_2; k_1 k_2} \overbrace{\boxed{\left(\sum_{n=1}^{R} J_{i_1 k_1}^n J_{i_2 k_2}^n \right)}}^{J_{i_1 i_2; k_1 k_2}} c_{i_1}^\dagger c_{k_1} c_{i_2}^\dagger c_{k_2}$$

Experimental couplings



Low rank SYK models

Low rank SYK models

$$\mathcal{L} = \frac{1}{2}\sum_{j=1}^{N}\psi_j\partial_\tau\psi_j - \frac{1}{2}\sum_{n=1}^{R} b_n b_n + (i)^{\frac{p}{2}}\sum_{n=1}^{R}\sum_{j_1<\cdots<j_p} J^n_{j_1\ldots j_p}\, b_n\psi_{j_1}\ldots\psi_{j_p}$$

$$\overline{J^n_{j_1\ldots j_p}} = 0 \qquad \overline{J^n_{j_1\ldots j_p} J^m_{k_1\ldots k_p}} = \frac{(p-1)!\,J^2}{N^p}\,\delta^{nm}\,\delta_{j_1k_1}\ldots\delta_{j_pk_p}$$

Already assumed that $R \sim N$

[Marcus, Vandoren 2018]
[Kim, Cao, Altman 2019]

Relation with $\mathcal{N} = 1$ supersymmetric SYK

$$Q = i \sum_{i<j<k} C_{ijk}\, \psi_i \psi_j \psi_k$$

$$H = Q^2 = E_0 + \sum_{i<j<k<l} J_{ijkl}\, \psi_i \psi_j \psi_k \psi_l$$

$$E_0 = \sum_{i<j<k} C_{ijk}^2 \qquad\qquad J_{ijkl} = -\frac{1}{2} \sum_a C_{aij} C_{akl}$$

compare with $\sum_{n=1}^{R} J^n_{i_1 k_1} J^n_{i_2 k_2}$

[Fu, Gaiotto, Maldacena, Sachdev 2016]

Bilocal operators

$$G(\tau_1,\tau_2) = \frac{1}{N}\sum_j \mathcal{T}\psi_j(\tau_1)\psi_j(\tau_2) \qquad D(\tau_1,\tau_2) = \frac{1}{R}\sum_n \mathcal{T} b_n(\tau_1) b_n(\tau_2)$$

$$1 = \int \mathcal{D}G\,\mathcal{D}\Sigma\,\exp\left[-\frac{N}{2}\Sigma(\tau_1,\tau_2)\left(G(\tau_1,\tau_2) - \frac{1}{N}\sum_j \psi_j(\tau_1)\psi_j(\tau_2)\right)\right]$$

$$1 = \int \mathcal{D}D\,\mathcal{D}\Pi\,\exp\left[-\frac{R}{2}\Pi(\tau_1,\tau_2)\left(D(\tau_1,\tau_2) - \frac{1}{R}\sum_n b_n(\tau_1) b_n(\tau_2)\right)\right]$$

Effective bilocal $G\Sigma D\Pi$ action

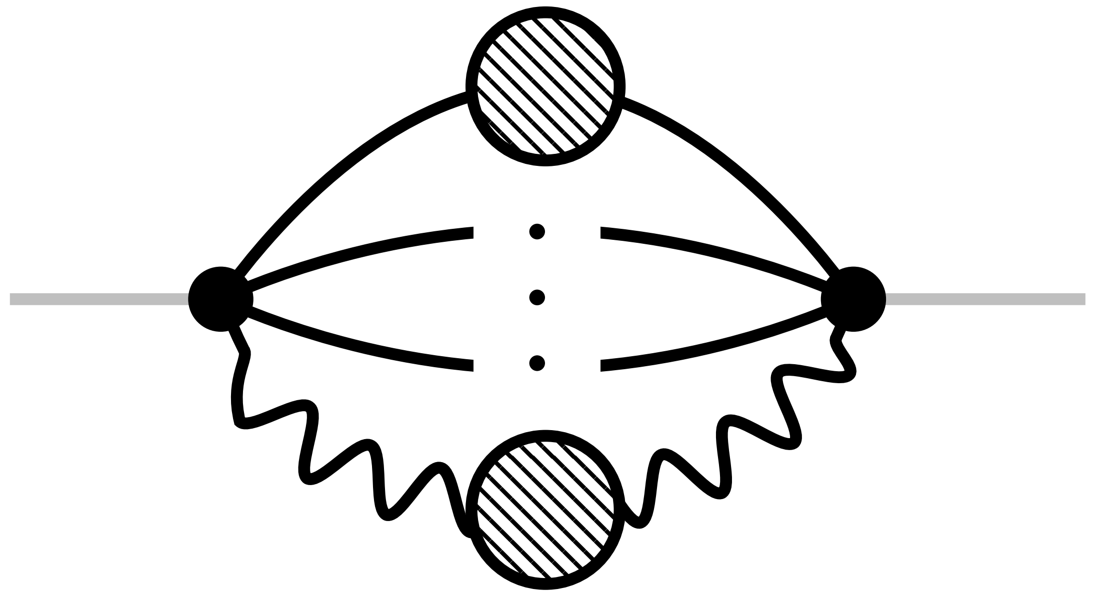
$$Z = \left\langle \int \mathcal{D}\psi\, \mathcal{D}b\, e^{-S[\psi,b]} \right\rangle_J = \int \mathcal{D}G\, \mathcal{D}\Sigma\, \mathcal{D}D\, \mathcal{D}\Pi\, e^{-S_{\text{eff}}[G,\Sigma,D,\Pi]}$$

$$\begin{aligned} S_{\text{eff}} = -\frac{N}{2}\operatorname{Tr}\log\Big(\partial_\tau - \Sigma(\tau)\Big) - \frac{R}{2}\operatorname{Tr}\log\Big(-1 - \Pi(\tau)\Big) + \frac{N}{2}\int \mathrm{d}\tau_1\mathrm{d}\tau_2\, G(\tau_1,\tau_2)\,\Sigma(\tau_1,\tau_2) \\ + \frac{R}{2}\int \mathrm{d}\tau_1\mathrm{d}\tau_2\, D(\tau_1,\tau_2)\,\Pi(\tau_1,\tau_2) - \frac{RJ^2}{2p}\int \mathrm{d}\tau_1\mathrm{d}\tau_2\, D(\tau_1,\tau_2)\, G^p(\tau_1,\tau_2) \end{aligned}$$

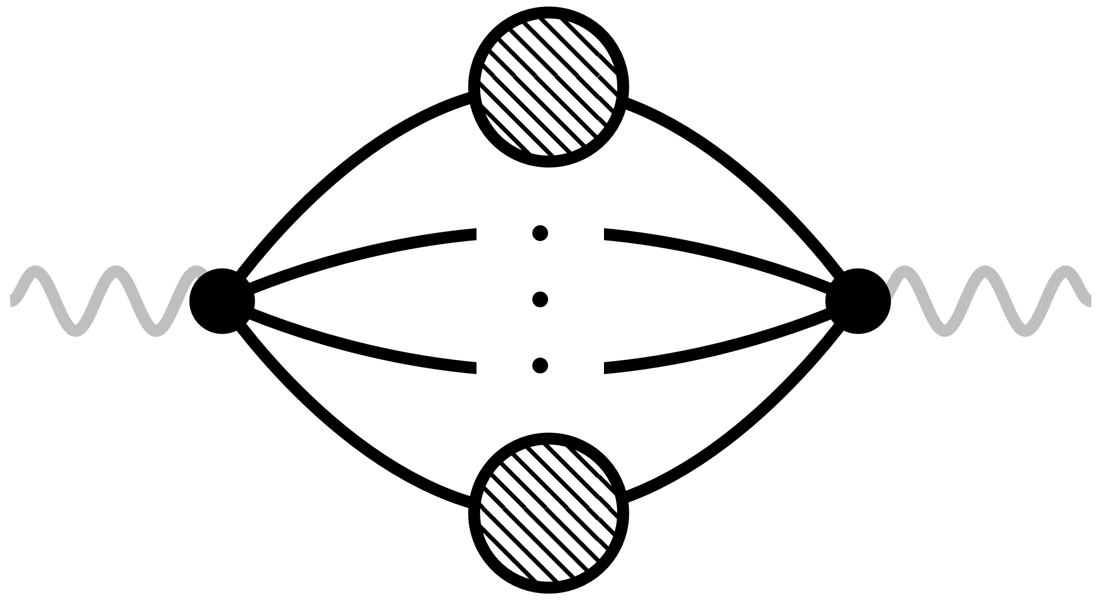
Schwinger-Dyson equations

$$\Sigma(\tau) = \gamma\, J^2\, G^{p-1}(\tau)\, D(\tau)$$

$$\gamma = R/N$$



$$\Pi(\tau) = \frac{J^2}{p}\, G^p(\tau)$$



$$G(i\omega) = \Big(-i\omega - \Sigma(i\omega)\Big)^{-1} \qquad D(i\omega) = \Big(-1 - \Pi(i\omega)\Big)^{-1}$$

Solution in the conformal limit

$$G(\tau) = A\,\mathrm{sgn}(\tau)\,\boxed{|\tau|^{-2\Delta}} \longrightarrow p\Delta + \Delta_b = 1$$

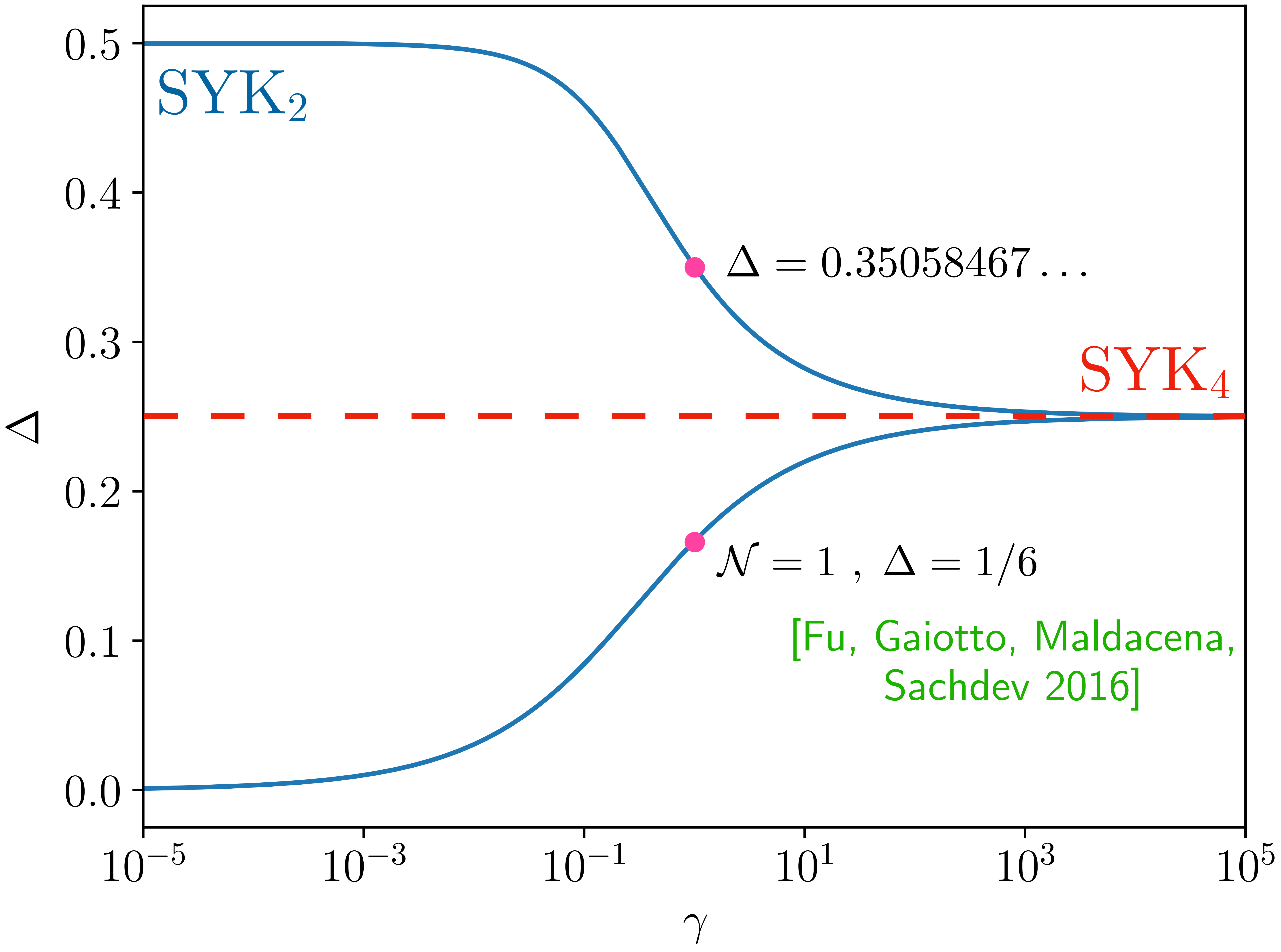
$$D(\tau) = -\frac{p\,\boxed{|\tau|^{2(p\Delta-1)}}}{4J^2A^p\sin^2(\pi p\Delta)\Gamma(1-2p\Delta)\Gamma(2p\Delta-1)}$$

$$\Sigma(\tau) = -\frac{\mathrm{sgn}(\tau)\,|\tau|^{-2(1-\Delta)}}{4A\cos^2(\pi\Delta)\Gamma(1-2\Delta)\Gamma(2\Delta-1)}$$

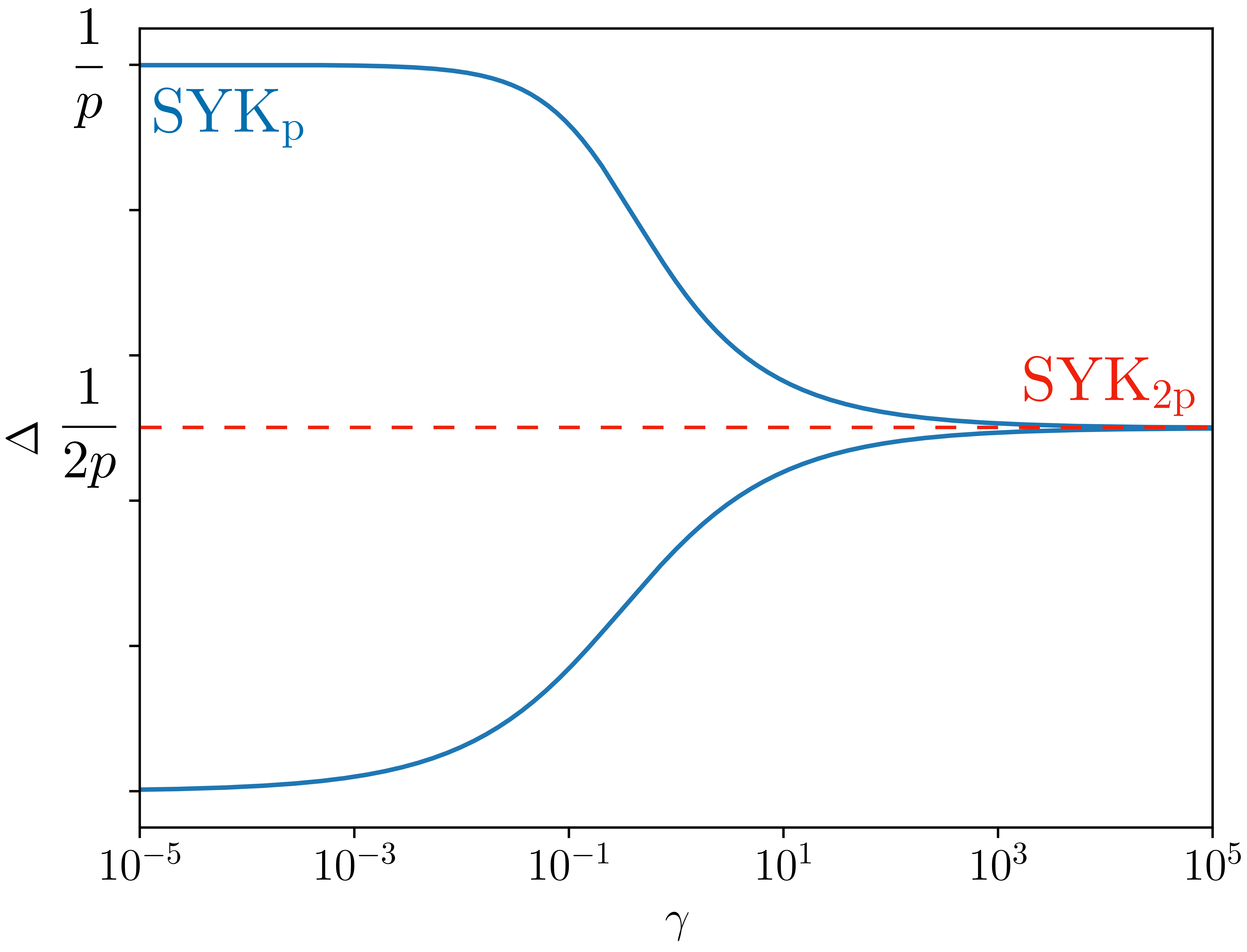
$$\Pi(\tau) = \frac{J^2A^p}{p}\,|\tau|^{-2p\Delta}$$

$$\boxed{\gamma = \frac{\sin^2(\pi p\Delta)}{p\cos^2(\pi\Delta)}\,\frac{\Gamma(1-2p\Delta)\Gamma(2p\Delta)}{\Gamma(1-2\Delta)\Gamma(2\Delta-1)}}$$

Scaling dimension



Scaling dimension (general p)



Solution at large p

$$G(\tau) = \frac{\text{sgn}(\tau)}{2}\left(1 + \frac{g(\tau)}{p} + \dots\right) \qquad D(\tau) = -\,\delta(\tau) + \frac{1}{2p}\,d(\tau) + \dots$$

$$\boxed{\partial_\tau^2\, g(\tau) = \mathcal{J}^2 e^{2g(\tau)}} \qquad \boxed{d(\tau) = \frac{\mathcal{J}}{\sqrt{\gamma}}\, e^{g(\tau)}}$$

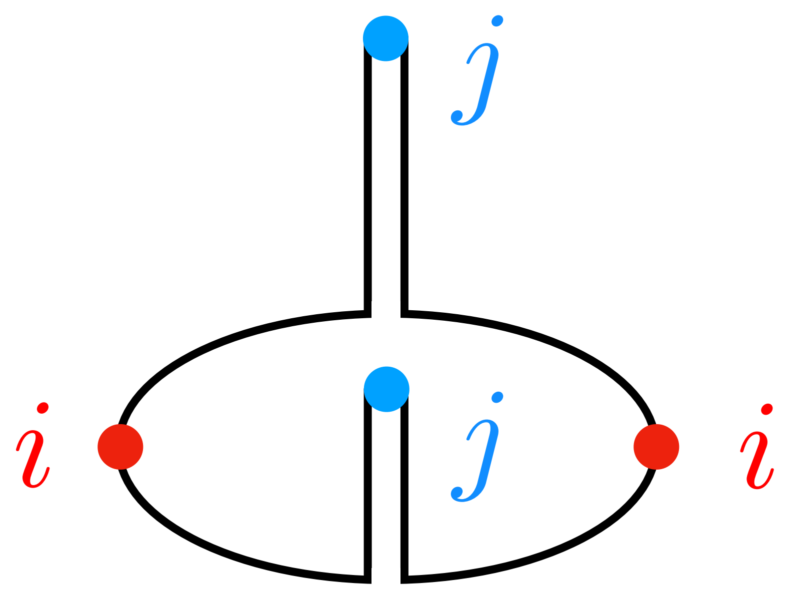
$$e^{g(\tau)} = \frac{\pi v}{\beta\mathcal{J}\cos\left[\frac{\pi v}{\beta}\left(\tau - \frac{\beta}{2}\right)\right]} \qquad \mathcal{J} \equiv \frac{\sqrt{\gamma}J^2}{2^{p-1}} \qquad \beta\mathcal{J} = \frac{\pi v}{\cos\left(\frac{\pi v}{2}\right)}$$

Lyapunov exponents

Out of Time Order Correlators

$$\mathcal{F}_{jiji}(t_1,t_2) = \frac{1}{Z(\beta)}\mathrm{Tr}\left[e^{-\beta H/4}\psi_j(t_1)e^{-\beta H/4}\psi_i(0)e^{-\beta H/4}\psi_j(t_2)e^{-\beta H/4}\psi_i(0)\right]$$

$$\mathcal{F}(t_1,t_2) = \frac{1}{N^2}\sum_{ij}\mathcal{F}_{jiji}(t_1,t_2)$$

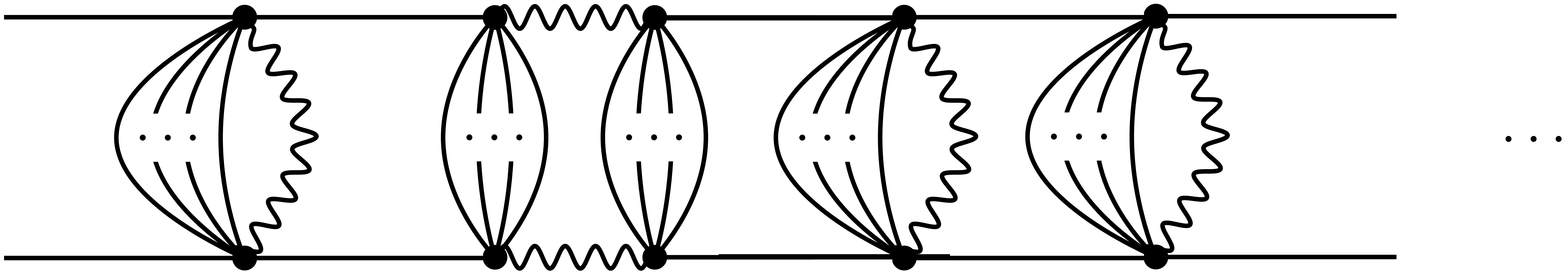


Connected diagrams

$$\mathcal{F}(t,t) = f_0 + \boxed{\frac{f_1}{N}\,e^{\frac{2\pi t}{\beta}}} + \dots$$

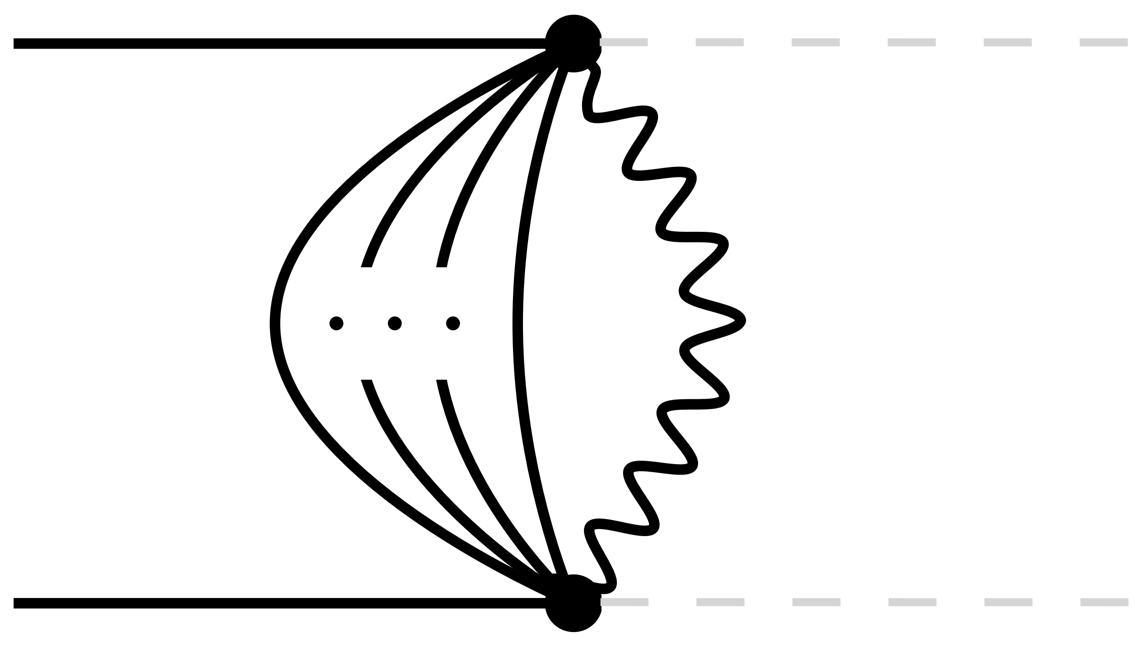
[Shenker, Stanford 2013] [Maldacena, Shenker, Stanford 2015]

Ladder diagrams



Bosonic kernel

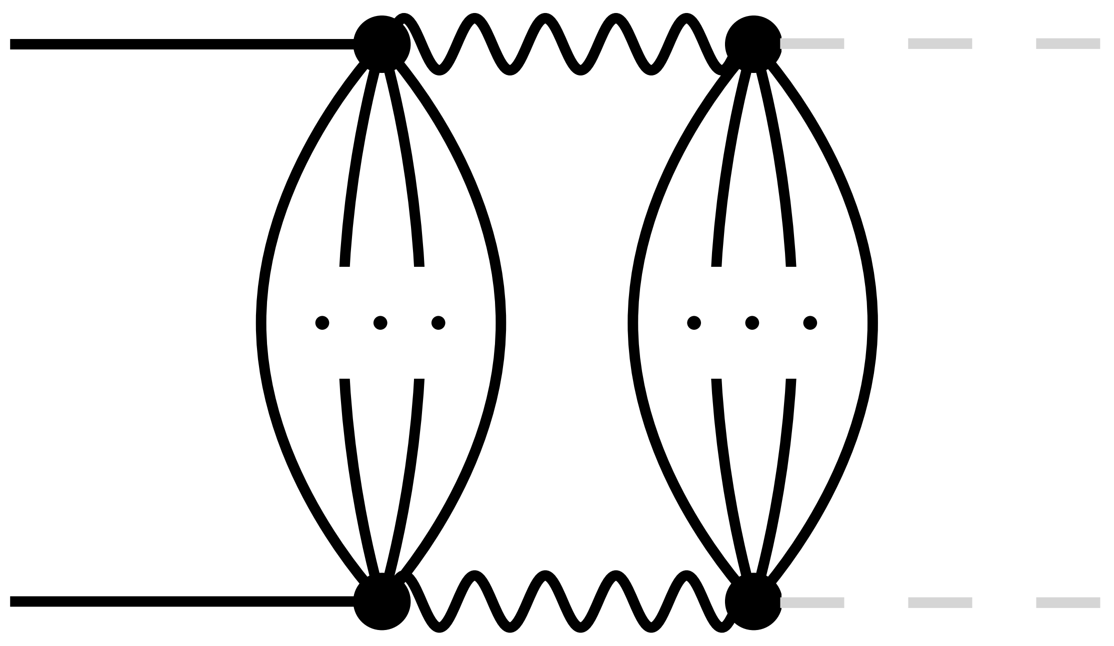
$$\mathcal{K}_b(t_1, t_2; t_3, t_4) \quad =$$



$$= \quad \gamma J^2 (p-1) G_R(t_{13}) G_R(t_{24}) D_W(t_{34}) G_W^{p-2}(t_{34})$$

Fermionic kernel

$$\mathcal{K}_f(t_1, t_2; t_3, t_4) \quad =$$



$$= \quad \gamma J^4 \int \mathrm{d}t_5 \, \mathrm{d}t_6 \, G_R(t_{15}) G_R(t_{26}) D_R(t_{53}) D_R(t_{64}) G_W^{p-1}(t_{34}) G_W^{p-1}(t_{56})$$

Eigenvalue equation for the kernel

$$\mathcal{K}(t_1, t_2; t_3, t_4) = \mathcal{K}_b(t_1, t_2; t_3, t_4) + \mathcal{K}_f(t_1, t_2; t_3, t_4)$$

Asymptotic growth: $$\mathcal{F}(t_1, t_2) = \int \mathrm{d}t_3 \, \mathrm{d}t_4 \, \mathcal{K}(t_1, t_2; t_3, t_4) \, \mathcal{F}(t_3, t_4)$$

[Kitaev 2015]

Real-time correlation functions in conformal limit

$$G_R(t) = 2A\cos(\pi\Delta)\theta(t)\left[\frac{\pi}{\beta\sinh(\frac{\pi t}{\beta})}\right]^{2\Delta} \qquad G_W(t) = A\left[\frac{\pi}{\beta\cosh(\frac{\pi t}{\beta})}\right]^{2\Delta}$$

$$D_R(t) = \frac{p\sin(\pi p\Delta)\theta(t)}{2J^2A^p\sin^2(\pi p\Delta)\Gamma(1-2p\Delta)\Gamma(2p\Delta-1)}\left[\frac{\pi}{\beta\sinh(\frac{\pi t}{\beta})}\right]^{2(1-p\Delta)}$$

$$D_W(t) = \frac{p}{4J^2A^p\sin^2(\pi p\Delta)\Gamma(1-2p\Delta)\Gamma(2p\Delta-1)}\left[\frac{\pi}{\beta\cosh(\frac{\pi t}{\beta})}\right]^{2(1-p\Delta)}$$

Conformal kernel eigenvalues

Ansatz: $$\mathcal{F}(t_1,t_2) = \boxed{e^{-h\frac{\pi(t_1+t_2)}{\beta}}} \cosh^{h-2\Delta}\left(\frac{\pi t_{12}}{\beta}\right)$$

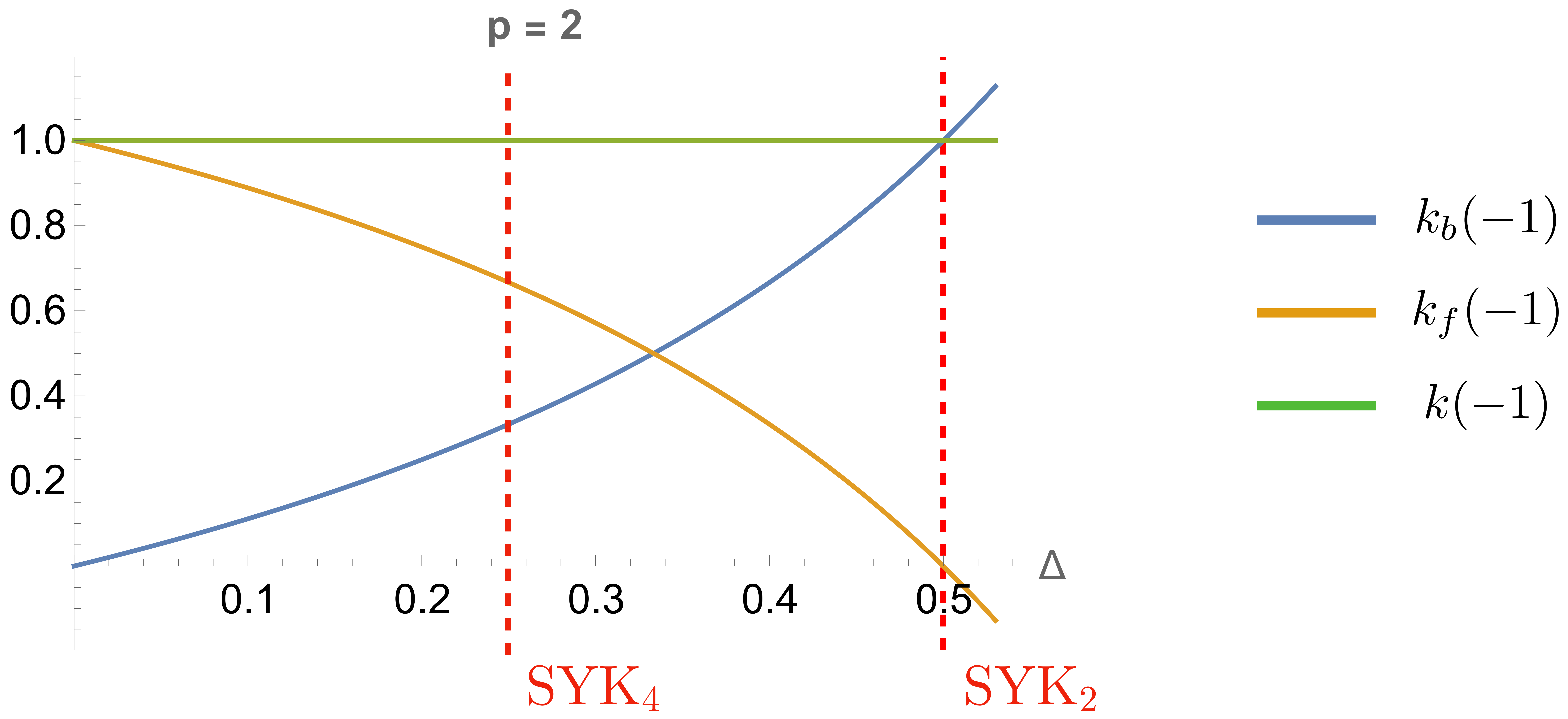
$h = -1$
maximal chaos

one finds $$k(h) = k_b(h) + k_f(h)$$

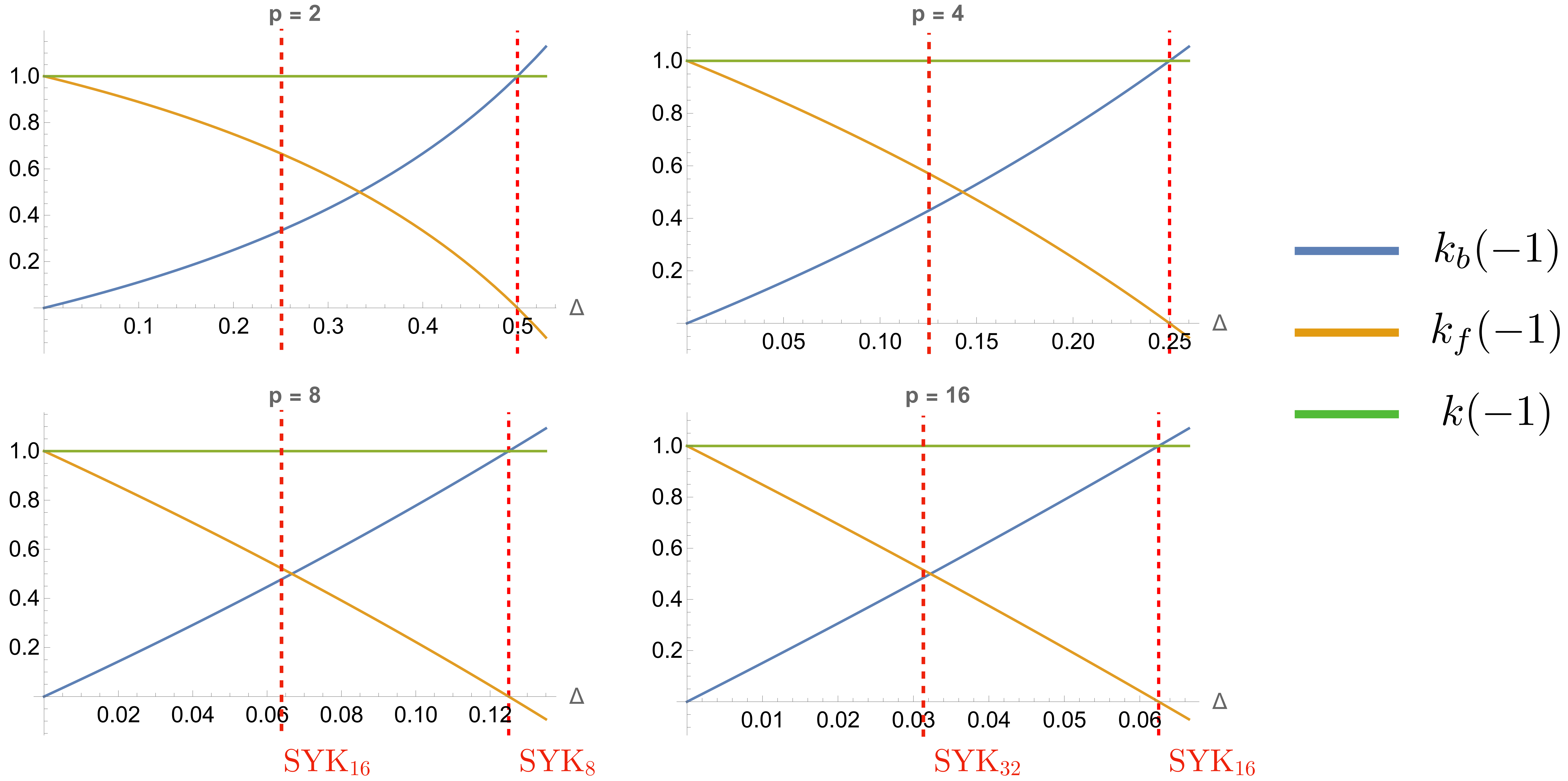
$$k_b(h) = (1-p)\,\frac{\Gamma(1-2\Delta)}{\Gamma(2\Delta-1)}\,\frac{\Gamma(2\Delta-h)}{\Gamma(2-2\Delta-h)}$$

$$k_f(h) = p\,\frac{\Gamma(1-2\Delta)}{\Gamma(2\Delta-1)}\,\frac{\Gamma(2p\Delta-1)}{\Gamma(1-2p\Delta)}\,\frac{\Gamma(2\Delta-h)}{\Gamma(2p\Delta-h)}\,\frac{\Gamma(2-2p\Delta-h)}{\Gamma(2-2\Delta-h)}$$

Conformal kernel eigenvalues



Kernel eigenvalues



Large p

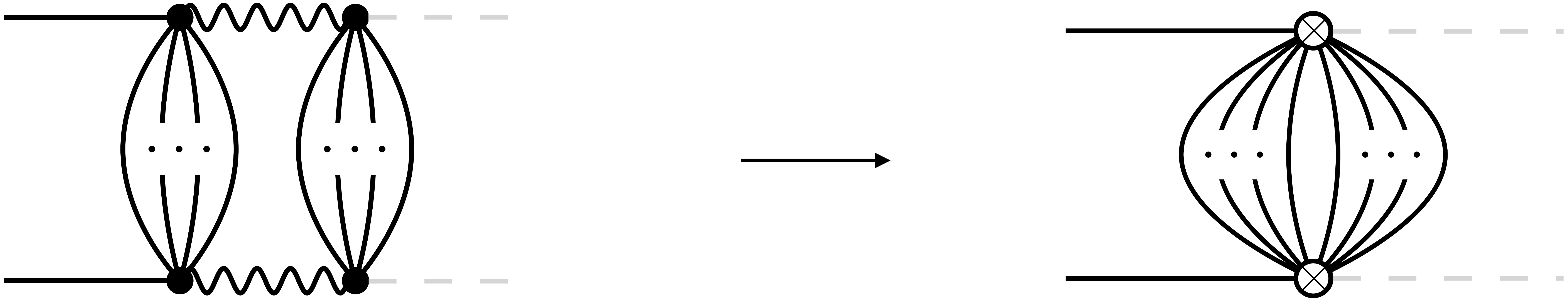
$$G_R(t_{12}) = \theta(t_{12}) \qquad\qquad D_R(t_{12}) = -\delta(t_{12})$$

$$G_W^{p-n}(t_{12}) = \frac{\pi v}{2^{p-n}\beta\mathcal{J}} \cosh^{-1}\left(\frac{\pi v t_{12}}{\beta}\right)$$

$$\boxed{D_W(t_{12}) = \frac{\pi v}{2p\beta\sqrt{\gamma}} \cosh^{-1}\left(\frac{\pi v t_{12}}{\beta}\right)}$$

$$\underbrace{\cancel{-\delta\left(t_{12} + \frac{i\beta}{2}\right)}}_{0} + \frac{1}{2p}\, d\left(t_{12} + \frac{i\beta}{2}\right)$$

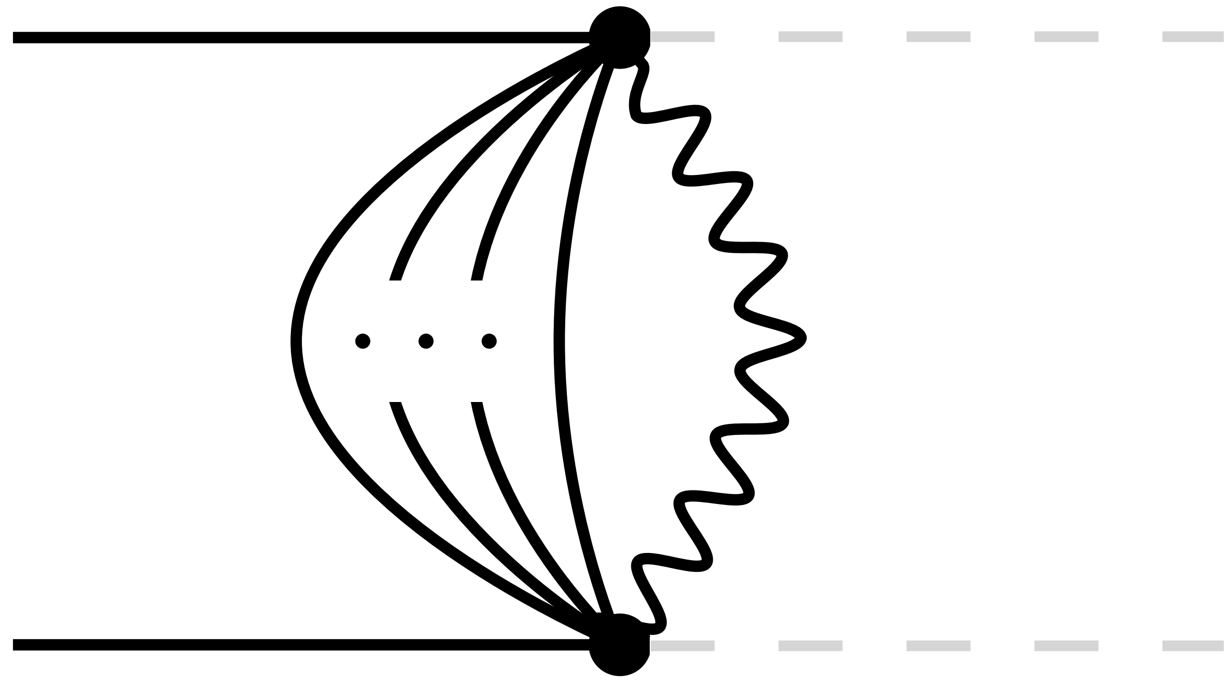
Ladder kernels at large p

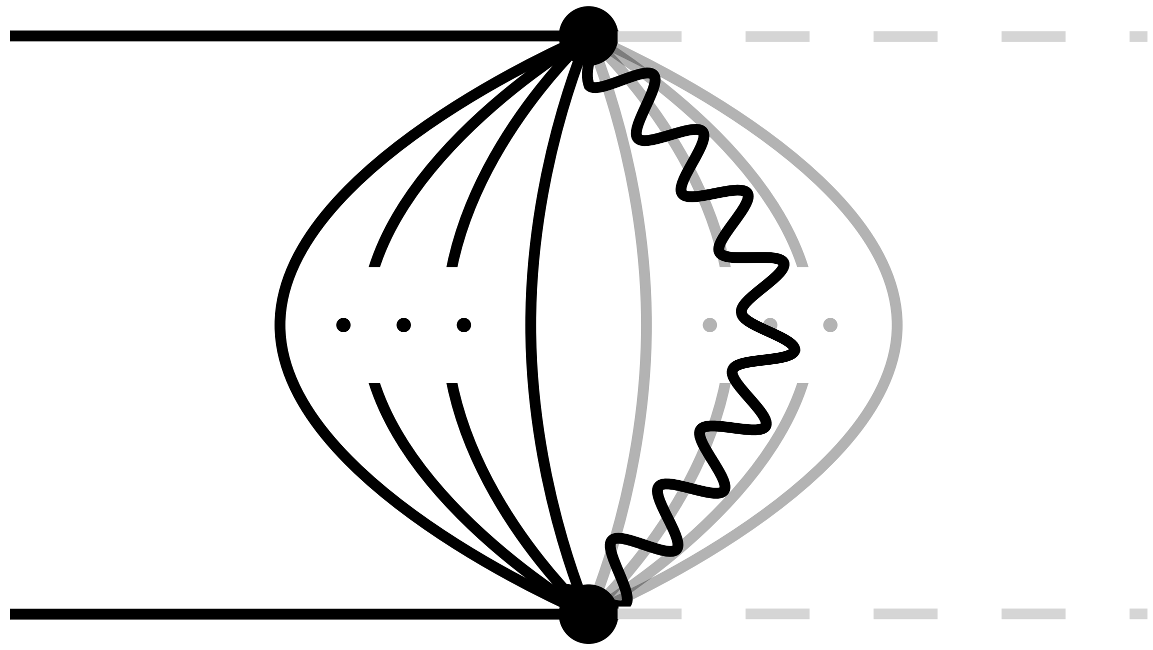


$$\gamma J^4 \int \mathrm{d}t_5\, \mathrm{d}t_6\, \mathrm{d}t_3\, \mathrm{d}t_4\, G_R(t_{15}) G_R(t_{26}) D_R(t_{53}) D_R(t_{64}) G_W^{p-1}(t_{34}) G_W^{p-1}(t_{56}) \mathcal{F}(t_3, t_4)$$

$$= \int \mathrm{d}t_3\, \mathrm{d}t_4\, G_R(t_{13}) G_R(t_{24})\, \frac{\pi^2 v^2}{\beta^2} \cosh^{-2}\left(\frac{\pi v\, t_{34}}{\beta}\right) \mathcal{F}(t_3, t_4)$$

Ladder kernels at large p





$$\gamma J^2 \cancel{(p-1)} \int \mathrm{d}t_3\, \mathrm{d}t_4\, G_R(t_{13}) G_R(t_{24}) \frac{1}{2\cancel{p}} d_W(t_{34}) G_W^{p-2}(t_{34})\, \mathcal{F}(t_3, t_4)$$

$$= \int \mathrm{d}t_3\, \mathrm{d}t_4\, G_R(t_{13}) G_R(t_{24})\, \frac{\pi^2 v^2}{\beta^2} \cosh^{-2}\left(\frac{\pi v\, t_{34}}{\beta}\right) \mathcal{F}(t_3, t_4)$$

OTOCs at large p

At large p the ladder kernel expectedly reduces to that of large p SYK.

The Lyapunov exponent is

$$\lambda = \frac{2\pi v}{\beta} \longrightarrow \begin{cases} \dfrac{2\pi}{\beta} & \text{for} \quad \beta J \gg 1 \\ 2\mathcal{J} & \text{for} \quad \beta J \ll 1 \end{cases}$$

Summary and future directions

Summary

- Shown how the (complex) SYK model can be realised in an optical cavity introducing disorder through light speckle patterns,
- Explored the physics of low-rank SYK models:
  - Interpolation $\mathrm{SYK_p} \longleftrightarrow \mathrm{SYK_{2p}}$ in conformal limit,
  - Recovered large-p SYK,
  - Studied Lyapunov exponents.

Future directions

“Experimental” future directions

- Dissipation *e.g.* random Lindbladians
- Monitoring the photons leaving the cavity
- Large temperatures of effective model: interesting?
- Realistic measurable quantities holographically meaningful?

Future directions

Theoretical future directions

- Double scaled limit?
  - $p \to \infty\,,\ N \to \infty\,,\ R \to \infty$ with $\lambda \equiv \dfrac{2p^2}{N}\,,\ \gamma \equiv \dfrac{R}{N}$ fixed.
- Schwarzian theory?
- Connections with physics of de Sitter?
  - $d(\tau)$ as a clock?

Thank you!

Backup slides

Converging couplings

$$Y = X_1 X_2 \longrightarrow P(Y)\,\mathrm{d}Y = \frac{1}{\pi\sigma^2}\,K_0\left(\frac{|Y|}{\sigma^2}\right)\,\mathrm{d}Y$$

$$\mathcal{Y} = \frac{1}{\sqrt{R}}\sum_{\alpha=1}^{R} Y_\alpha$$

$$\downarrow$$

$$P(\mathcal{Y})\,\mathrm{d}\mathcal{Y} = \frac{\sqrt{R}}{\sqrt{\pi}\,\sigma^2\,\Gamma(R/2)}\left(\frac{\sqrt{R}\,|\mathcal{Y}|}{2\sigma^2}\right)^{\frac{R-1}{2}} K_{\frac{1-R}{2}}\left(\frac{\sqrt{R}\,|\mathcal{Y}|}{\sigma^2}\right)\,\mathrm{d}\mathcal{Y}$$

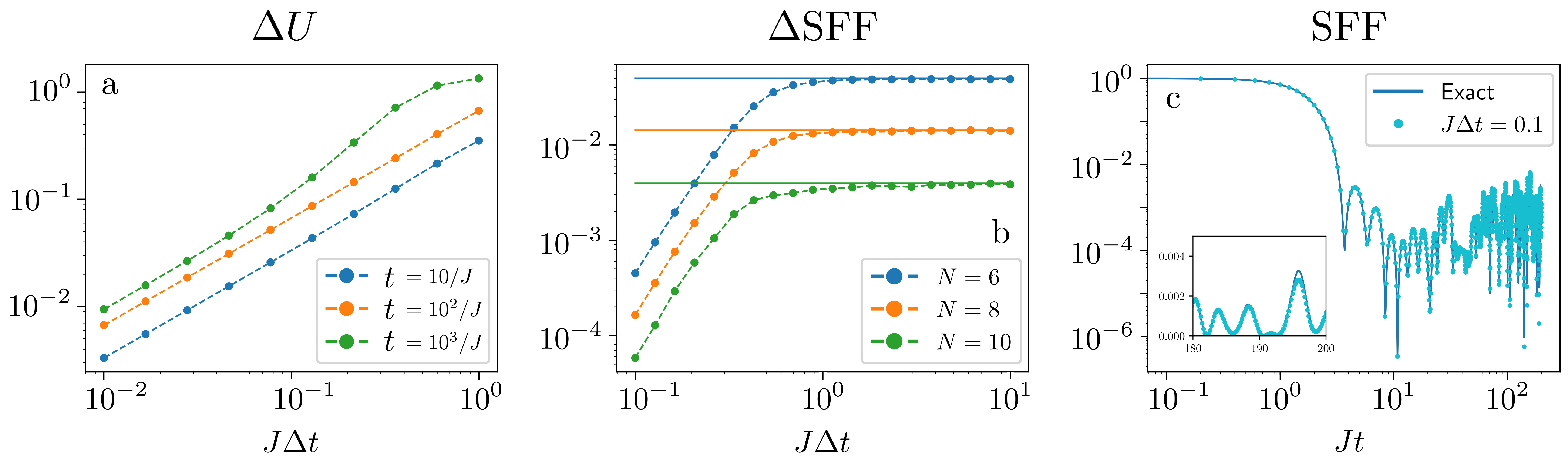
Converging couplings

$$P(\mathcal{Y})\,\mathrm{d}\mathcal{Y} = \frac{\sqrt{R}}{\sqrt{\pi}\,\sigma^2\,\Gamma(R/2)}\left(\frac{\sqrt{R}\,|\mathcal{Y}|}{2\sigma^2}\right)^{\frac{R-1}{2}} K_{\frac{1-R}{2}}\left(\frac{\sqrt{R}\,|\mathcal{Y}|}{\sigma^2}\right)\mathrm{d}\mathcal{Y}$$

$$= \frac{e^{-\frac{\mathcal{Y}^2}{2\sigma^4}}}{\sqrt{2\pi\sigma^4}} + \frac{1}{R}\left(\frac{3}{4} - \frac{3\mathcal{Y}^2}{2\sigma^4} + \frac{\mathcal{Y}^4}{4\sigma^8}\right)\frac{e^{-\frac{\mathcal{Y}^2}{2\sigma^4}}}{\sqrt{2\pi\sigma^4}}$$

$$+\frac{1}{R^2}\left(\frac{25}{32} - \frac{45\mathcal{Y}^2}{8\sigma^4} + \frac{65\mathcal{Y}^4}{16\sigma^8} - \frac{17\mathcal{Y}^6}{24\sigma^{12}} + \frac{\mathcal{Y}^8}{32\sigma^{16}}\right)\frac{e^{-\frac{\mathcal{Y}^2}{2\sigma^4}}}{\sqrt{2\pi\sigma^4}} + \mathcal{O}(R^{-3})$$

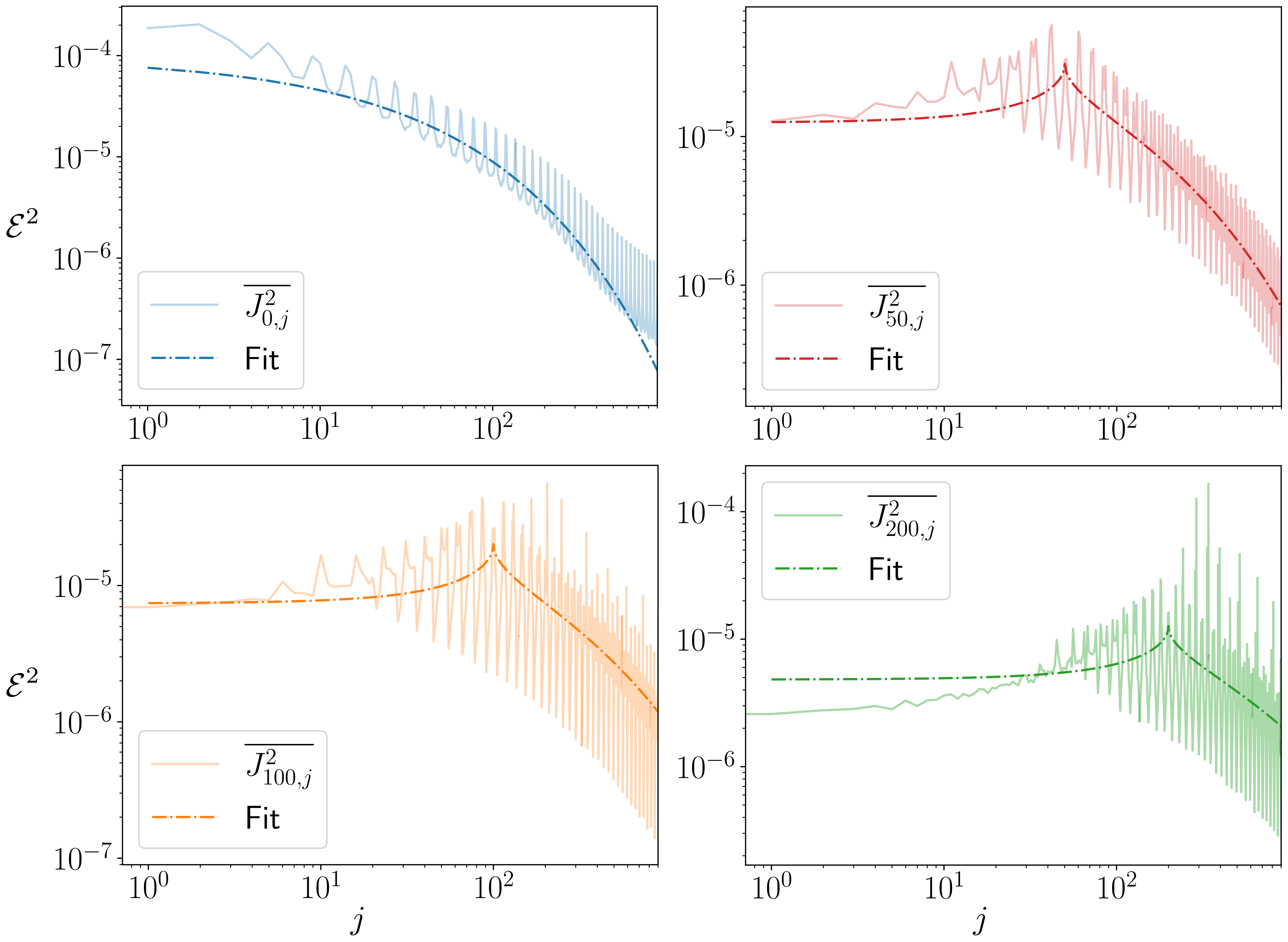
Bounds on trotterization error



$$U_{\text{eff}}(T) \equiv e^{-iH_{\text{eff}}t} = \left(\prod_{\alpha=1}^{R} e^{-iH_\alpha t/n}\right)^n + \frac{t^2}{2n}\sum_{\alpha<\beta}^{R}[H_\alpha, H_\beta] + \ldots$$

$$\overline{\left\| \sum_{\alpha<\beta}[H_\alpha, H_\beta]\right\|^2} \lesssim 2\times 10^2\,\frac{J^4R^2}{N^2} + \mathcal{O}(R^2/N^3)$$

Dynamical determination of N



$$\overline{J^2_{ij}} \sim \exp\left(\left|\frac{\sqrt{i}-\sqrt{j}}{\sqrt{N_{\text{eff}}}}\right|\right)$$

i	$r = 0.06$	$r = 0.15$
0	$N \approx 20$	$N \approx 120$
50	$N \approx 60$	$N \approx 190$
100	$N \approx 100$	$N \approx 280$
200	$N \approx 210$	$N \approx 440$

How many R's?

$$R = \gamma N^{\alpha}$$

$$\alpha < 1 \longrightarrow \begin{cases} T > T_{\mathrm{c}} & \text{Free fermions + no condensation} \\ T \leq T_{\mathrm{c}} & \text{SYK}_{\mathrm{p}} \text{ + boson condensation} \end{cases}$$

$$\boxed{\alpha = 1} \longrightarrow \text{Interpolation SYK}_{\mathrm{p}} \text{ to SYK}_{2\mathrm{p}} \text{ + no condensation}$$

$$\alpha > 1 \longrightarrow \text{SYK}_{2\mathrm{p}} \text{ + no condensation}$$

[Kim, Cao, Altman 2019]