# Simulating the Sachdev-Ye-Kitaev model in cQED platforms

Works with R. Baumgartner*, S. Bandyopadhyay, F. Orsi, N. Sauerwein, P. Hauke, J.P. Brantut and J. Sonner within the **HologrAPh consortium**

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# Holography at 30

[Maldacena 1997]

In its first 30 years, the study of holographic dualities has had a significant impact on many areas of physics, including of course

- String theory and quantum gravity
- Strongly coupled QFTs and gauge theories
- Black holes, the origin of the B-H entropy and the black hole interior

but also, quite remarkably, on

- Quantum information
- Non-equilibrium physics
- Quantum chaos and integrability…

This suggests that many important lessons remain to be discovered.

# Holographic systems in the lab

Realizing a quantum system with a holographic gravity dual is rather challenging, as it involves

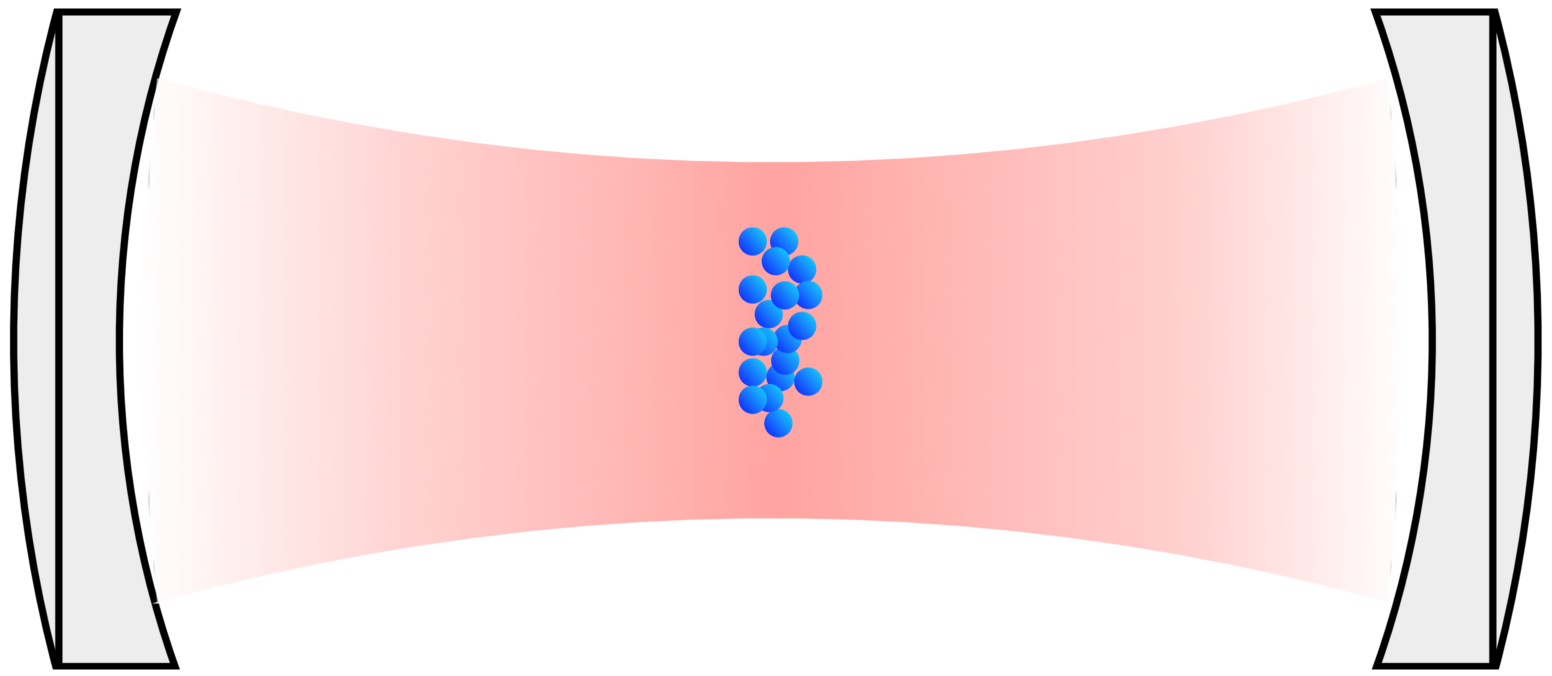
- Large number of degrees of freedom
- Strong coupling, in order to have Einstein-type weakly coupled bulk gravity
- Precise control over interactions, to generate symmetry algebra or at least disorder
- When it reduces to a quantum mechanical system, all-to-all connectivity

One of the simplest systems to realize is the Sachdev-Ye-Kitaev model [Sachdev, Ye 1993] [Kitaev 2015] [Sachdev 2015]

$$H = (i)^{\frac{p}{2}} \sum_{j_1 < \cdots < j_p} J_{j_1 \ldots j_p} \psi_{j_1} \ldots \psi_{j_p}$$

# Why cavity QED?

Optical cavities are arrangements of mirrors that confine light waves.



Optical cavities in the QED regime naturally realize several of the ingredients listed before:

- Large number of degrees of freedom
- Strong coupling
- Precise control over interactions
- All-to-all connectivity

Depend on *number of atoms* and *tunable parameters* (large number of degrees of freedom, strong coupling, precise control over interactions)

Mediated by *cavity photons* (precise control over interactions, all-to-all connectivity)

See also: [Danshita et al. 2016], p-adic AdS/CFT [Bentsen et al. 2019], tensor networks [Swingle's talk], …

# Complex, Yukawa & low rank SYK models

Complex SYK: $$H = \sum_{ij;kl} J_{ij;kl}\, c_i^\dagger c_j^\dagger c_k c_l$$

[Sachdev 2015]

Yukawa SYK: $$H = \sum_{n=1}^{R} \Delta_n a_n^\dagger a_n + \sum_{n=1}^{R} \sum_{ij} J_{ij}^n c_i^\dagger c_j (a_n + a_n^\dagger)$$

[Esterlis, Schmalian 2019]

$\Delta \gg J$ ↓

(complex) low rank SYK: $$H = -\sum_{n=1}^{R} \frac{1}{\Delta_n} \sum_{ij;kl} J_{ij}^n J_{kl}^n c_i^\dagger c_j c_k^\dagger c_l$$

[Marcus, Vandoren 2018]
[Kim, Cao, Altman 2019]

Main part of the talk: *how to realize this model(s) in a cQED setup?*

# HologrAPh consortium

J.P. Brantut

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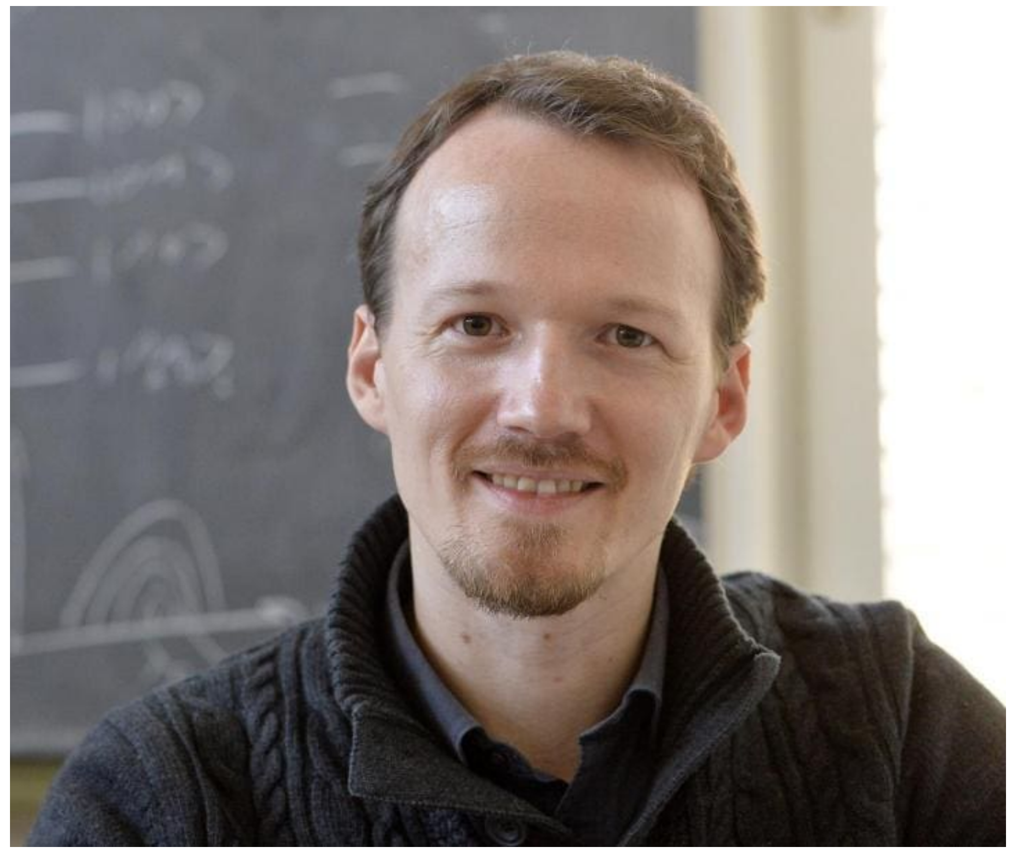

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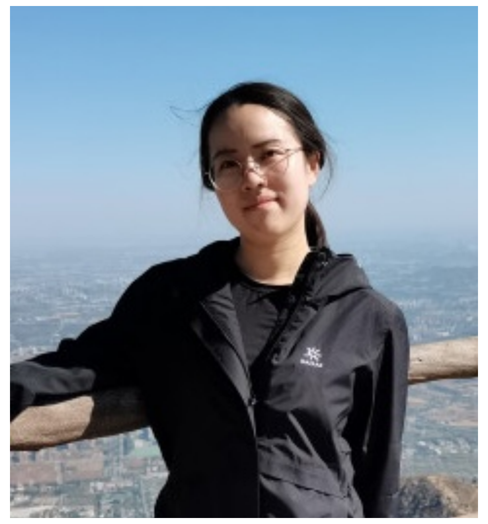
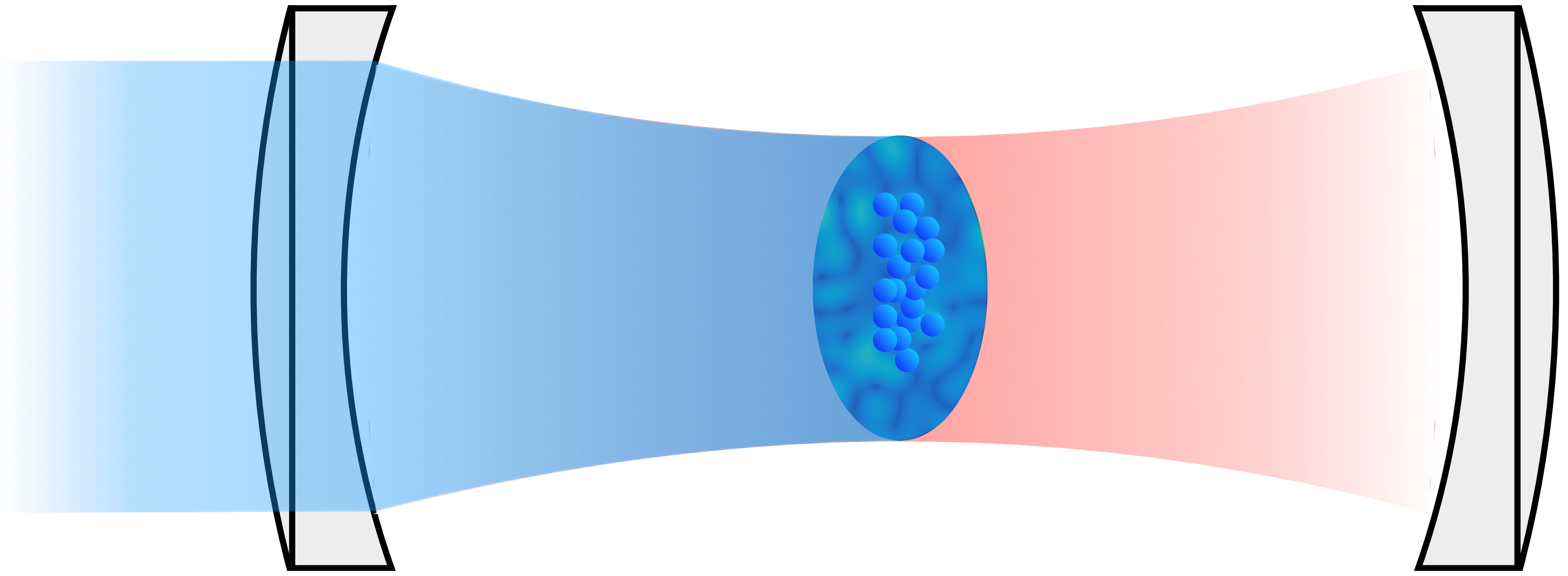



F. Orsi, E. Fedotova, S. Bandyopadhyay, A. Windey, D. P. Solis, A. Legramandi, R. Baumgartner, Y.-N. Zhou, R. Löwenberg + . . .

[Uhrich et al. 2023]
[Baumgartner et al. 2024 & 2025]

# Cavity QED setup



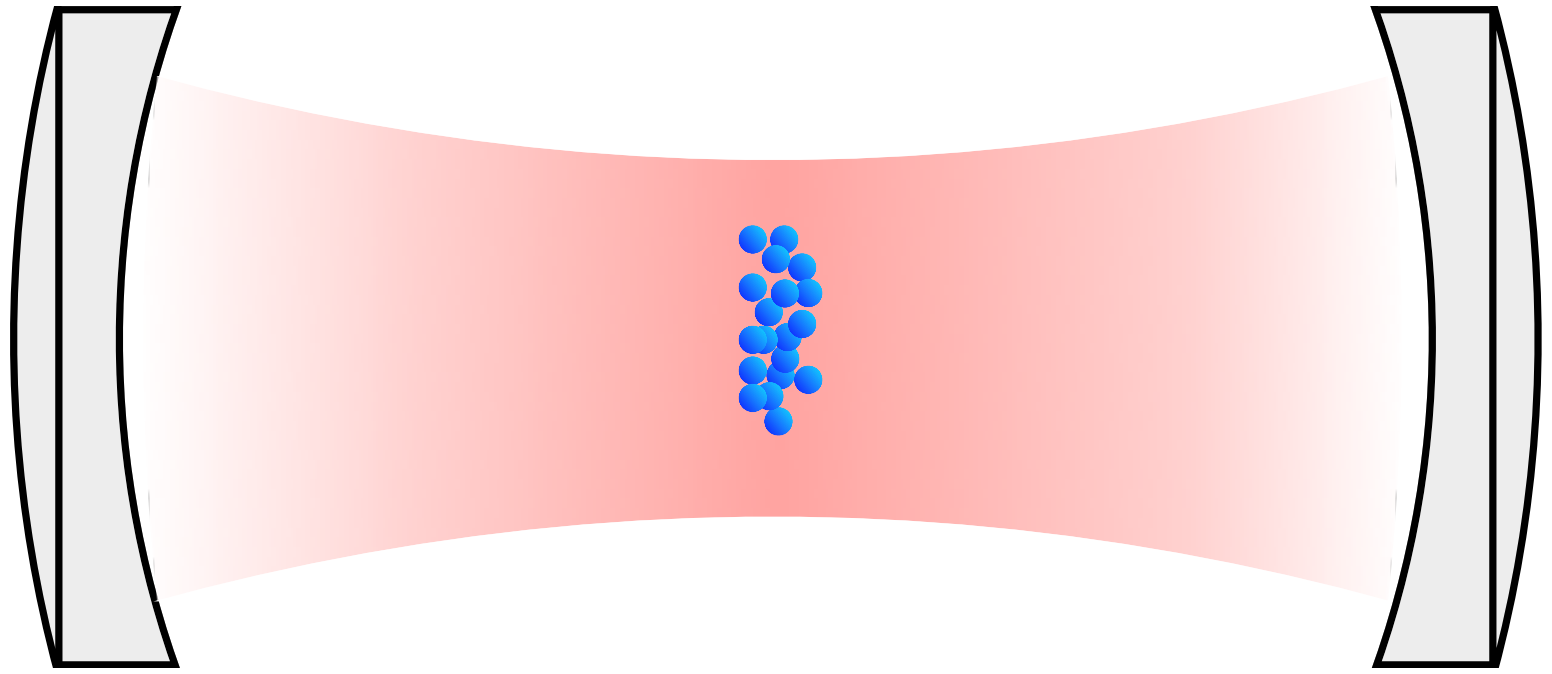
$$H = H_{\mathrm{kt}} + H_{\mathrm{c}} + H_{\mathrm{a}} + H_{\mathrm{ac}} + H_{\mathrm{ad}}$$

Several ingredients:

- Harmonic trap
- Cavity mode
- Li-6 atoms
- QED interaction
- Pump drive

[Uhrich et al. 2023]
[Baumgartner et al. 2024 & 2025]

# Cavity QED setup



$$H = H_{\mathrm{kt}} + H_{\mathrm{c}} + H_{\mathrm{a}} + H_{\mathrm{ac}} + H_{\mathrm{ad}}$$

Several ingredients:

- Harmonic trap
- Cavity mode
- Li-6 atoms
- QED interaction
- Pump drive

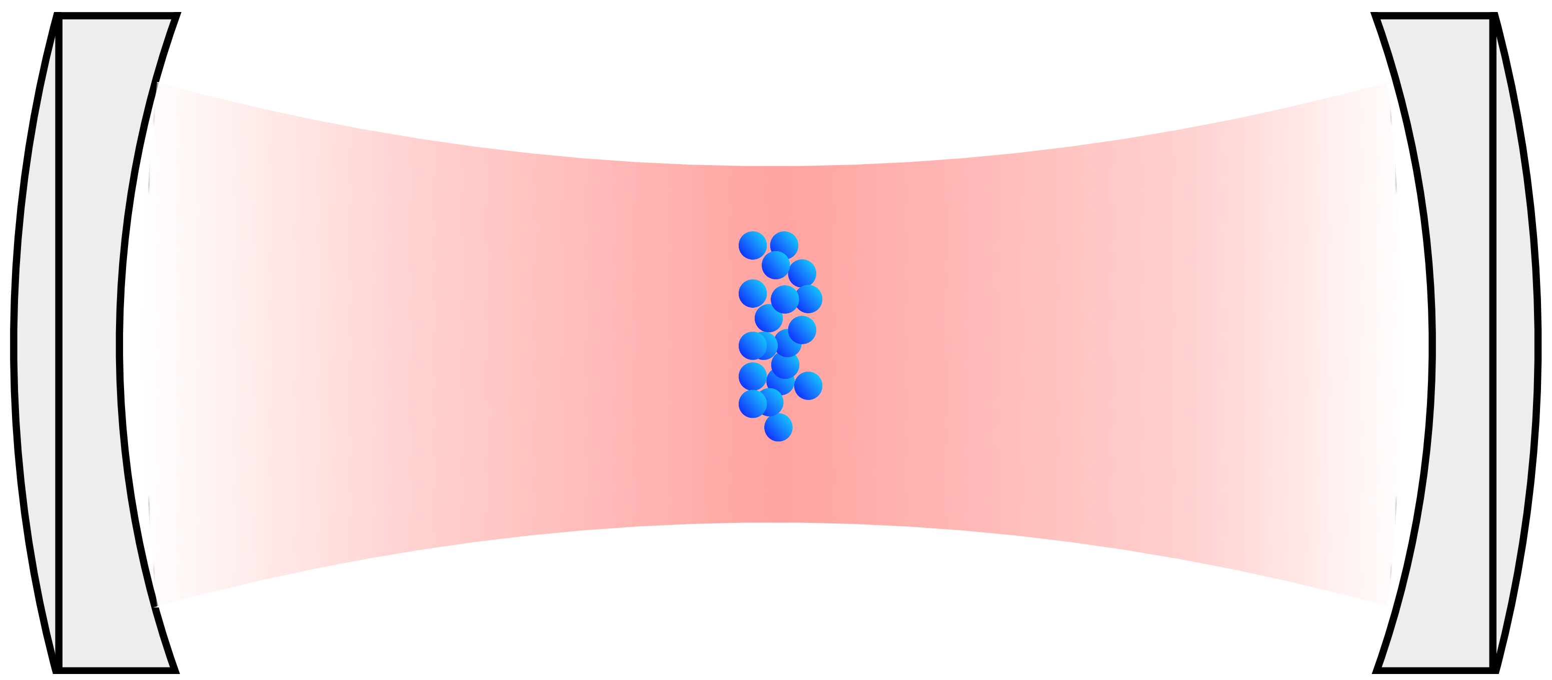
$$H_{\mathrm{kt}} = \sum_{\mathrm{s}\in\{\mathrm{e,g}\}} \int \mathrm{d}^2 r\, \psi_{\mathrm{s}}^\dagger(r) \left( \frac{p^2}{2m_{\mathrm{at}}} + V_{\mathrm{t}}(r) \right) \psi_{\mathrm{s}}(r)$$

$$\psi_{\mathrm{g}}(r) = \sum_i \phi_i(r)\, c_i$$

Very shallow trap. The $i$ 's will be SYK sites

# Cavity QED setup

[Uhrich et al. 2023]
[Baumgartner et al. 2024 & 2025]



$$H = H_{\mathrm{kt}} + H_{\mathrm{c}} + H_{\mathrm{a}} + H_{\mathrm{ac}} + H_{\mathrm{ad}}$$

Several ingredients:

- Harmonic trap
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- Pump drive

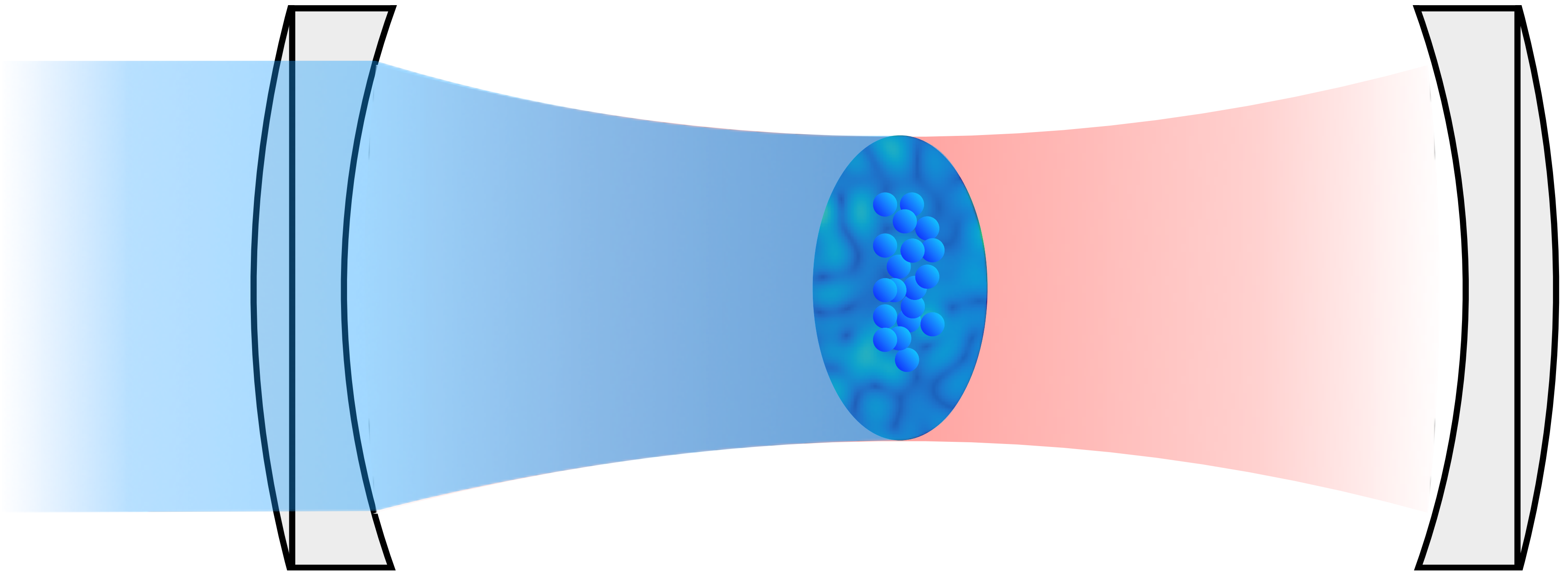
$$H_{\mathrm{c}} = \omega_{\mathrm{c}}\, a^\dagger a$$

Different regimes can be considered:

| | |
|---|---|
| Multimode | [Uhrich et al. 2023] |
| Single-mode | [Baumgartner et al. 2024 & 2025] |

[Uhrich et al. 2023]
[Baumgartner et al. 2024 & 2025]

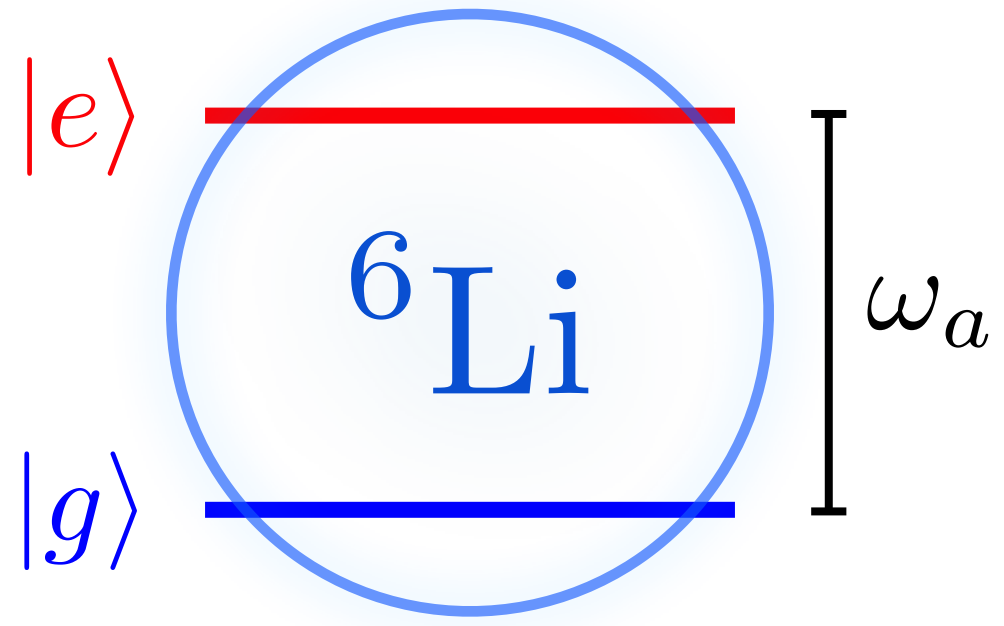
# Cavity QED setup



$$H = H_{\mathrm{kt}} + H_{\mathrm{c}} + H_{\mathrm{a}} + H_{\mathrm{ac}} + H_{\mathrm{ad}}$$

Several ingredients:

- Harmonic trap
- Cavity mode
- Li-6 atoms
- QED interaction
- Pump drive

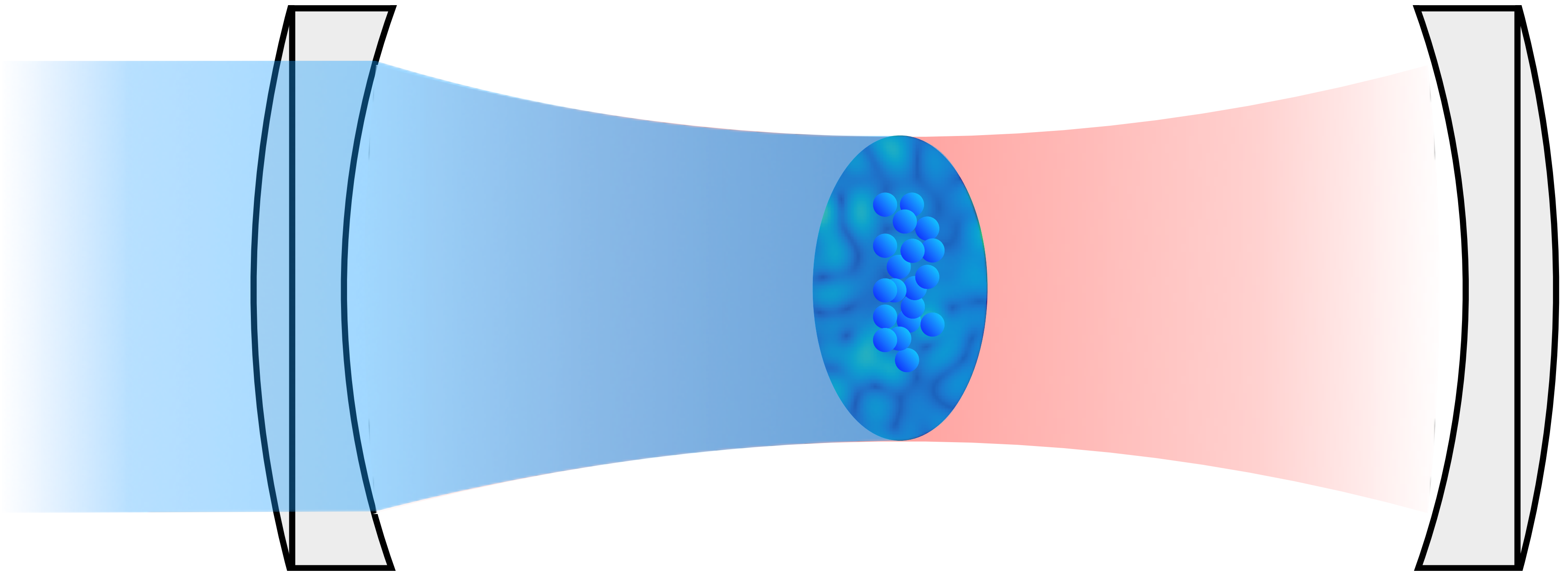



$$H_{\mathrm{a}} = \int \mathrm{d}^2 r\, \omega_{\mathrm{a}}(r)\, \psi_{\mathrm{e}}^{\dagger}(r)\, \psi_{\mathrm{e}}(r)$$

Speckle pattern
(additional blue light)

# Cavity QED setup

[Uhrich et al. 2023]
[Baumgartner et al. 2024 & 2025]

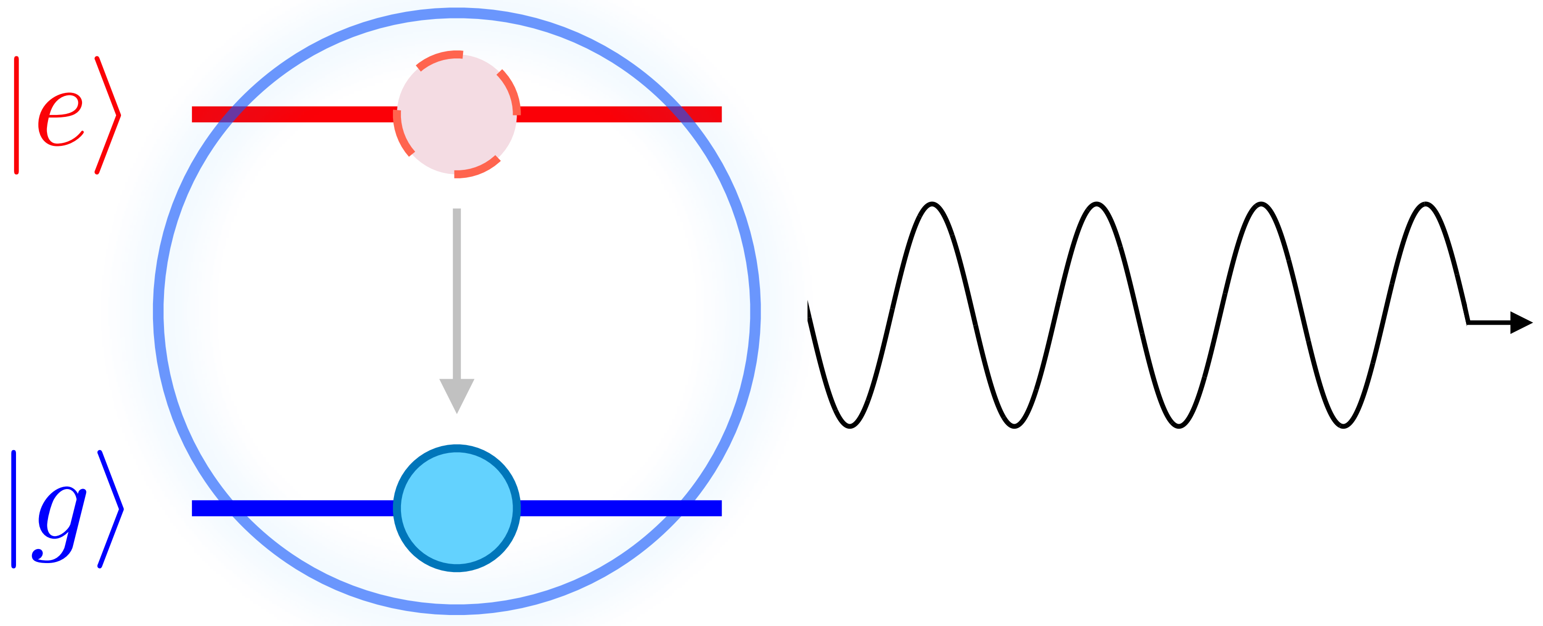


$$H = H_{\mathrm{kt}} + H_{\mathrm{c}} + H_{\mathrm{a}} + H_{\mathrm{ac}} + H_{\mathrm{ad}}$$

Several ingredients:

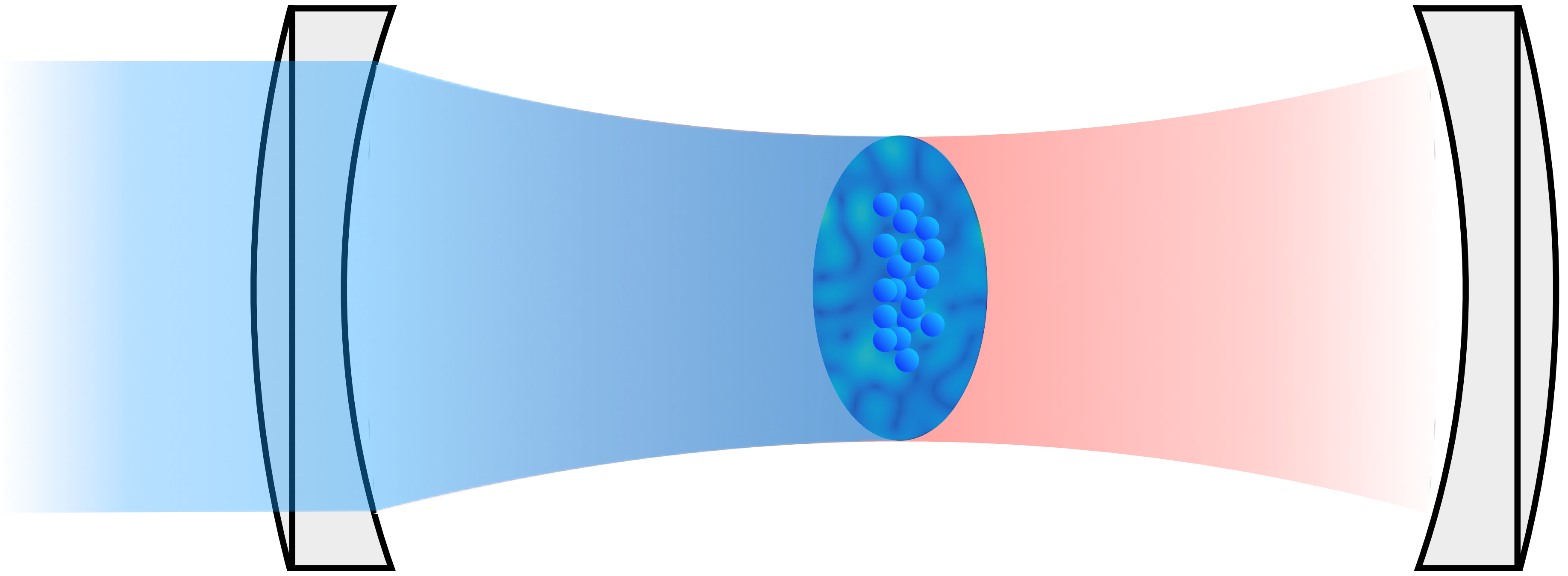
- Harmonic trap
- Cavity mode
- Li-6 atoms
- QED interaction
- Pump drive

$$H_{\mathrm{ac}} = \frac{\Omega_{\mathrm{c}}}{2} \int \mathrm{d}^2 r \, \left( g_{\mathrm{c}}(r) \psi_{\mathrm{e}}^{\dagger}(r) \psi_{\mathrm{g}}(r) a + \mathrm{h.c.} \right)$$



# Cavity QED setup

[Uhrich et al. 2023]
[Baumgartner et al. 2024 & 2025]



$$H = H_{\mathrm{kt}} + H_{\mathrm{c}} + H_{\mathrm{a}} + H_{\mathrm{ac}} + H_{\mathrm{ad}}$$

Several ingredients:

- Harmonic trap
- Cavity mode
- Li-6 atoms
- QED interaction
- Pump drive

$$H_{\mathrm{ad}} = \Omega_{\mathrm{d}} \int \mathrm{d}^2 r \, \left( g_{\mathrm{d}}(r)\, e^{-i\omega_{\mathrm{d}} t} \psi_{\mathrm{e}}^{\dagger}(r) \psi_{\mathrm{g}}(r) + \mathrm{h.c.} \right)$$

Going to rotating frame: measuring frequencies wrt $\omega_d$

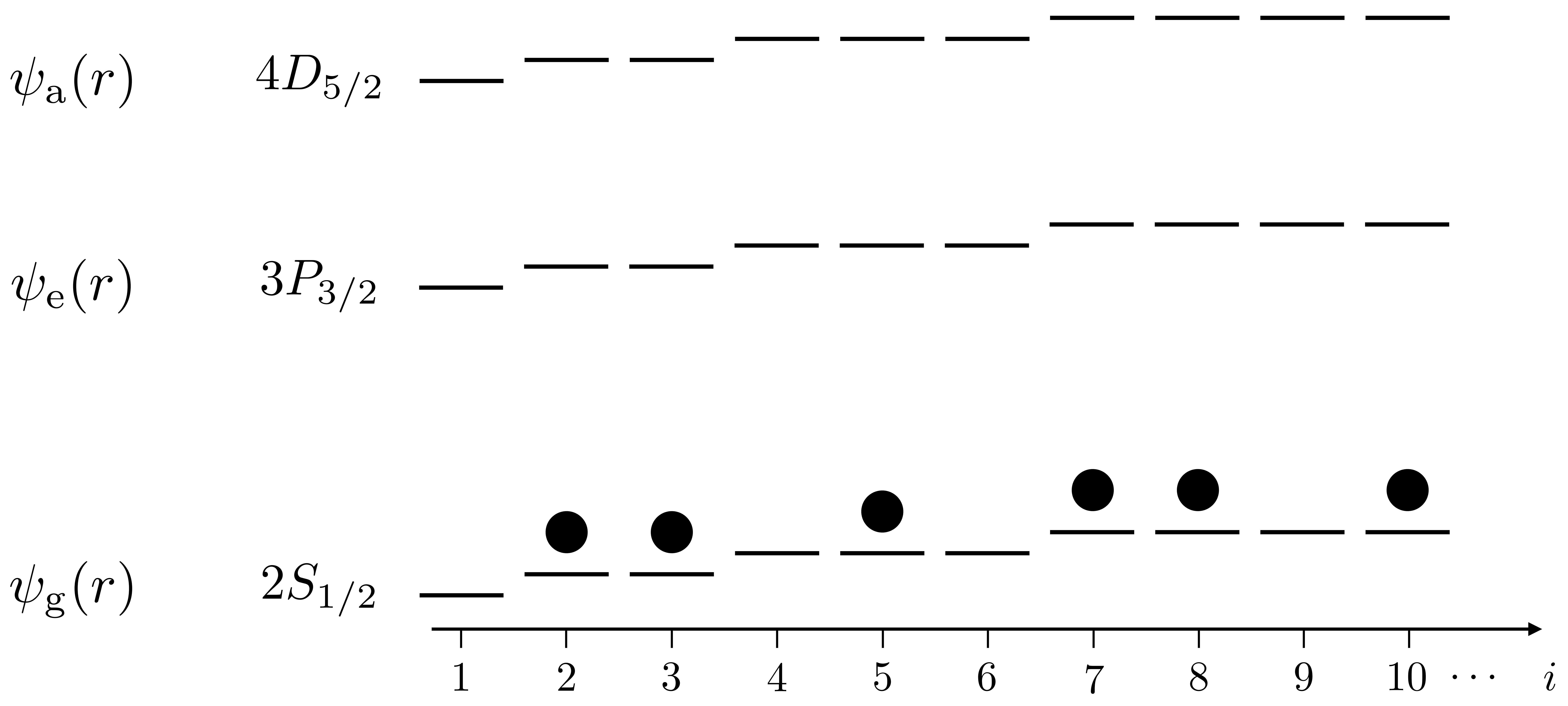
$$\left.\begin{array}{l} \omega_{\mathrm{c}} \rightarrow \Delta_{\mathrm{cd}} \equiv \omega_{\mathrm{c}} - \omega_{\mathrm{d}} \\ \omega_{\mathrm{a}} \rightarrow \Delta_{\mathrm{ad}} \equiv \omega_{\mathrm{a}} - \omega_{\mathrm{d}} \end{array}\right\} \longleftarrow$$

Detunings

(largest energy scale)

# The full picture

[Uhrich et al. 2023]
[Baumgartner et al. 2024 & 2025]
See also [Sauerwein et al. 2023]



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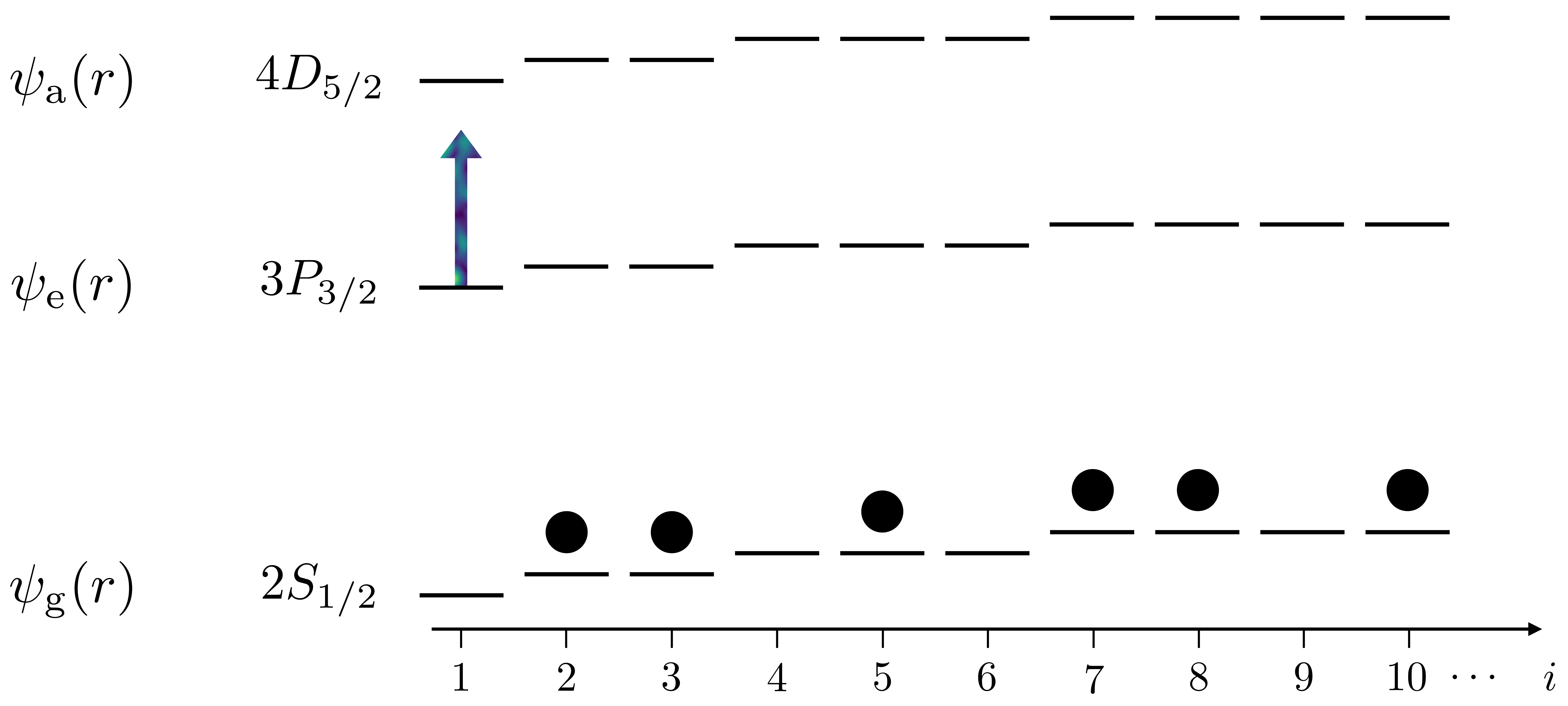
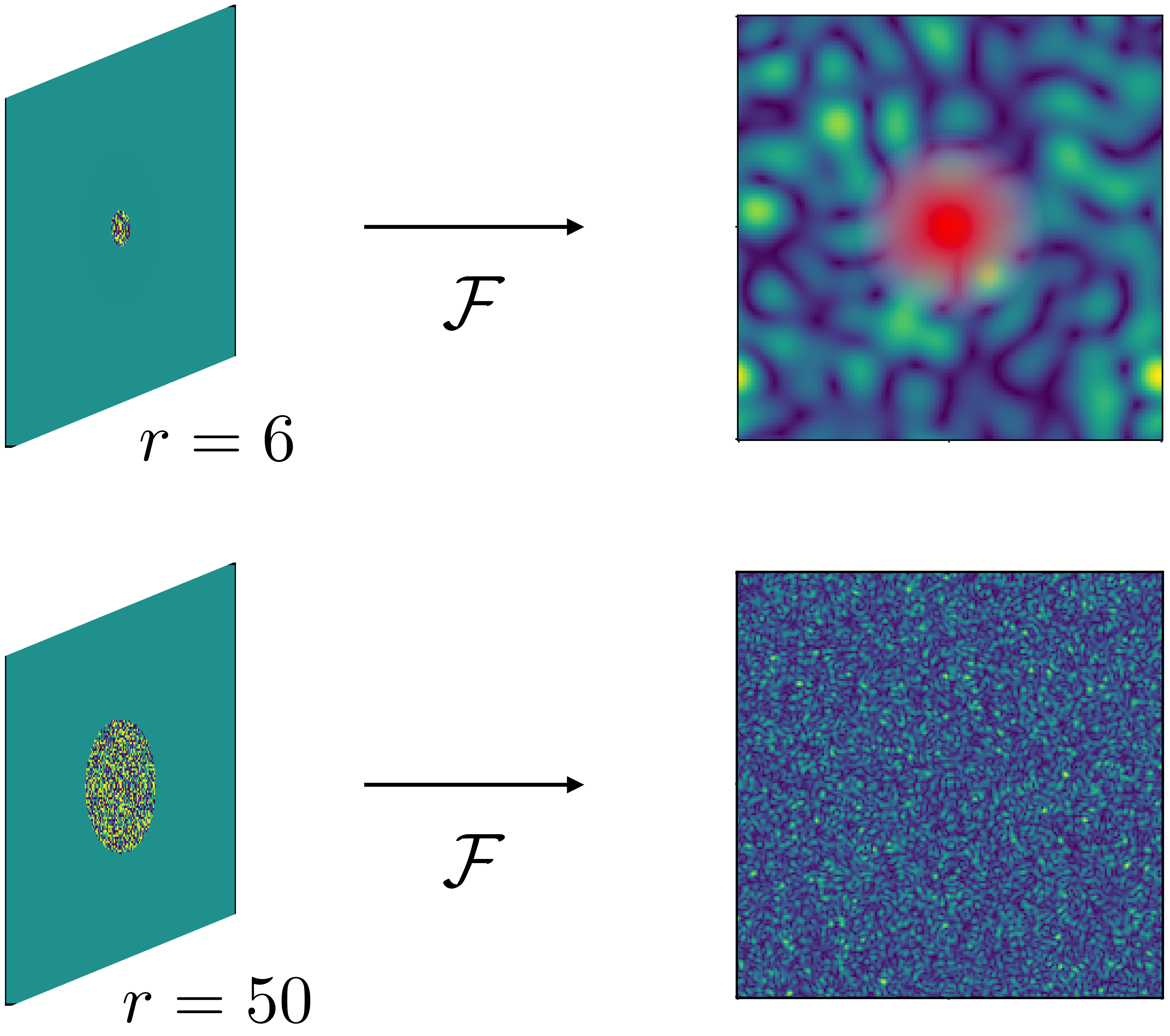


# Speckle patterns

Disorder in the additional blue light is obtained through speckle patterns generated via Spatial Light Modulator.

Numerically, it can be implemented via a Fourier Transform of a random-phase mask.

Red halo represents the physical size of the harmonic trap's fundamental length.



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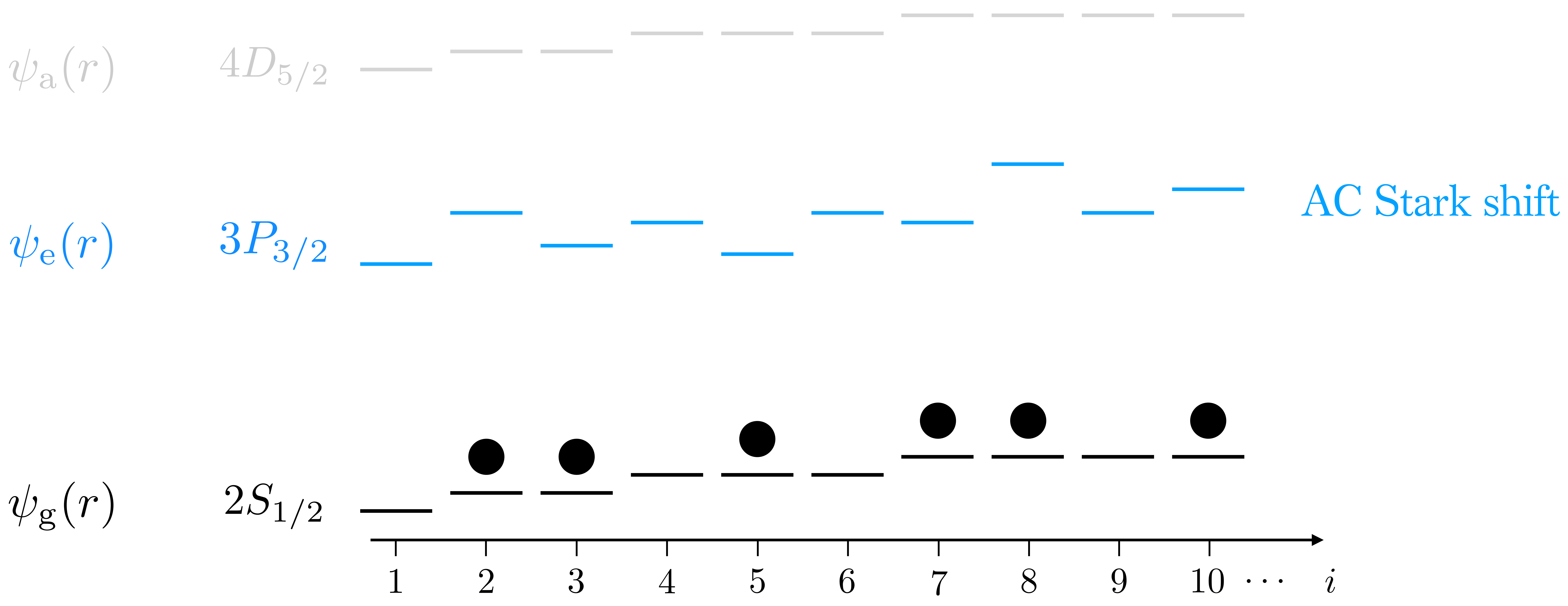

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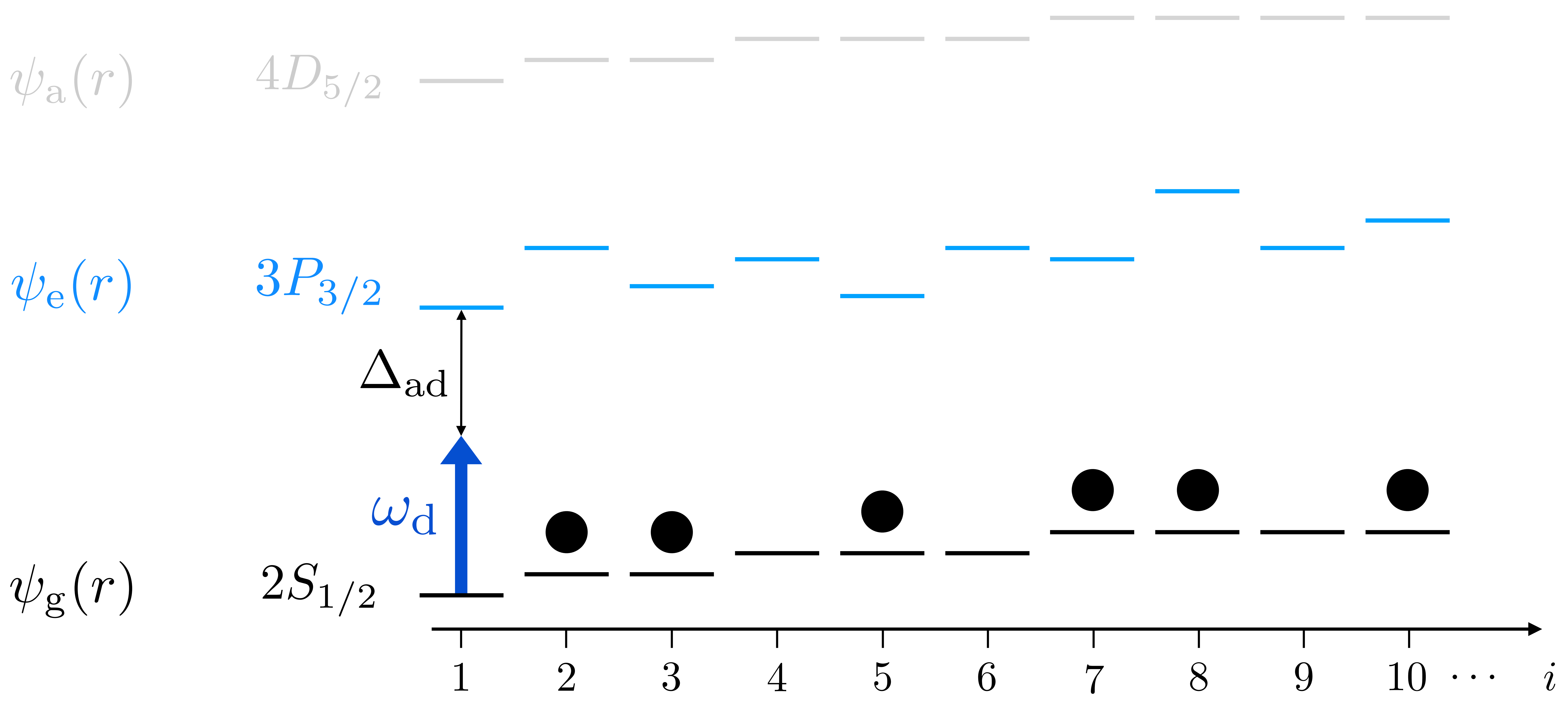

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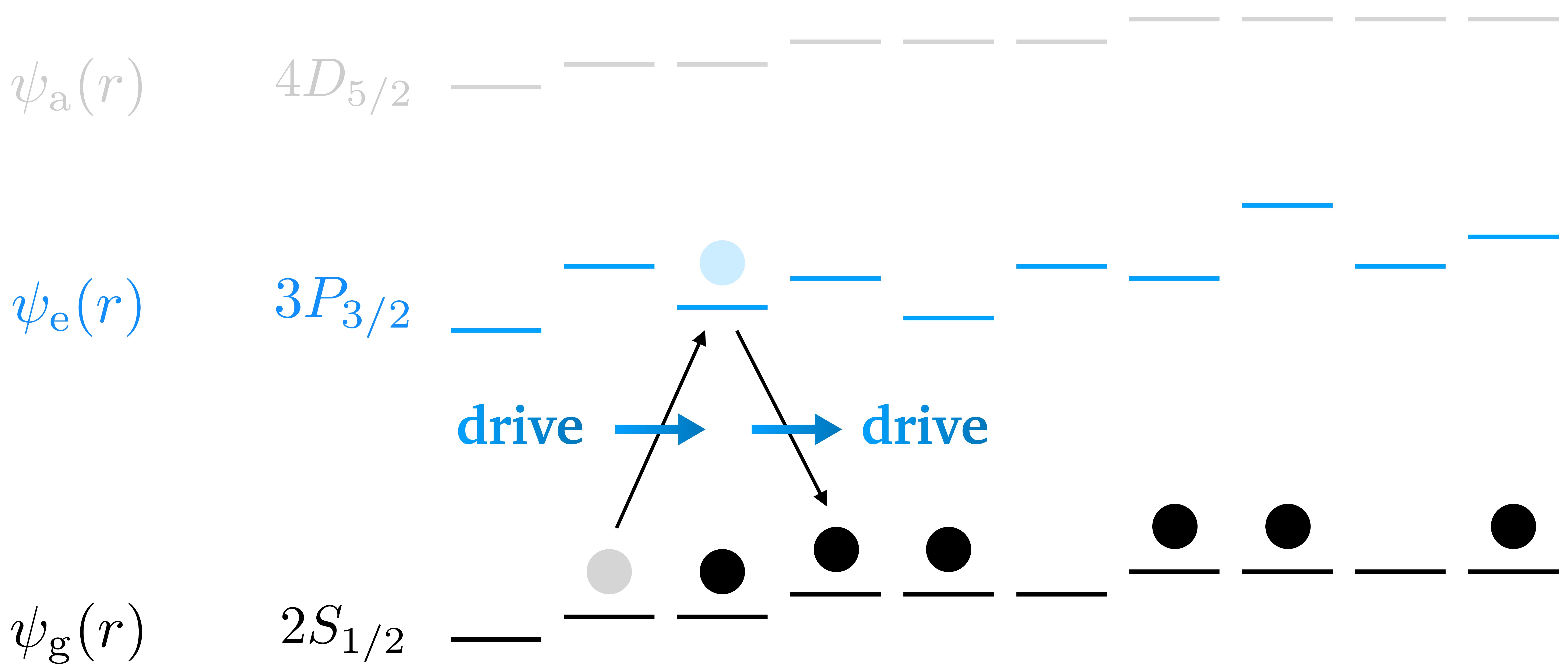
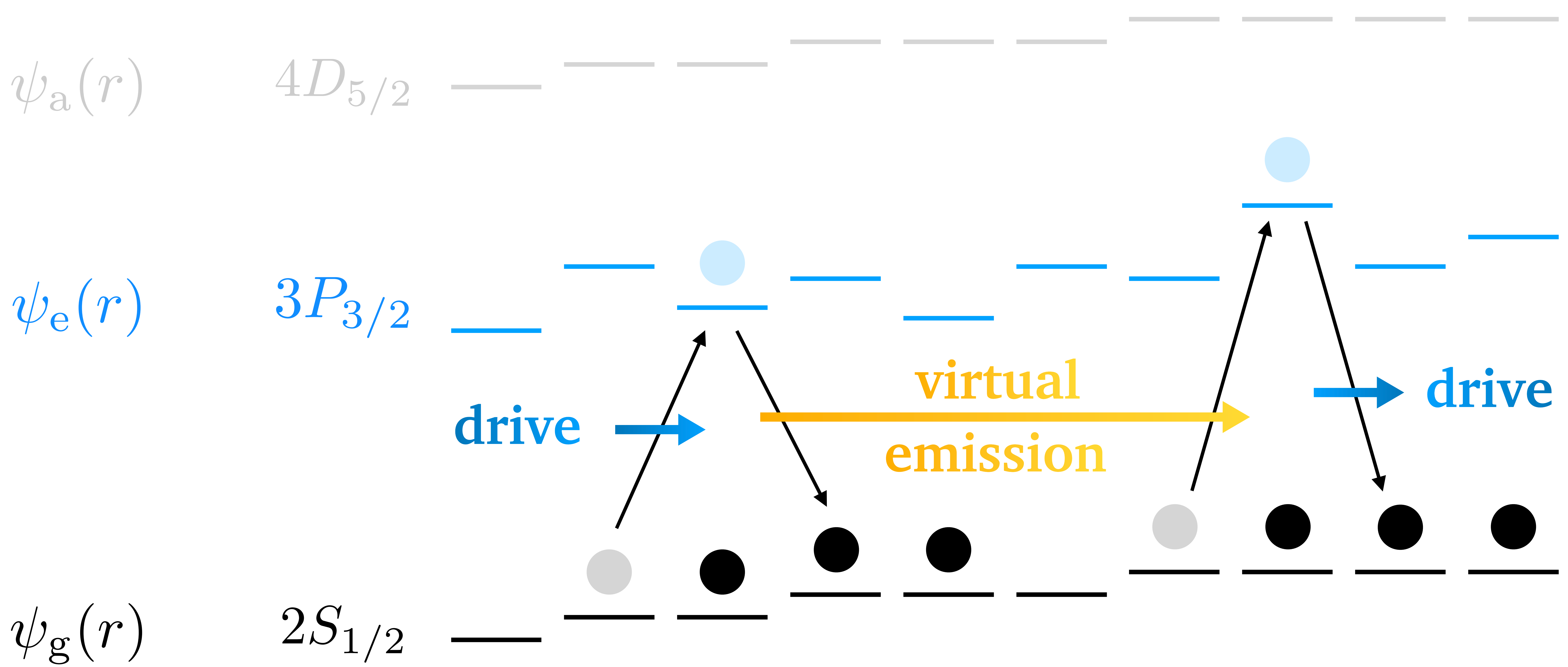



Interaction term: $\frac{\Omega_{\mathrm{d}}^2}{\Delta_{\mathrm{ad}}} \int \mathrm{d}^2 r \, \frac{\Delta_{\mathrm{ad}}}{\Delta_{\mathrm{ad}}(r)} \, \psi_{\mathrm{g}}^{\dagger}(r) \psi_{\mathrm{g}}(r)$

# The full picture

[Uhrich et al. 2023]
[Baumgartner et al. 2024 & 2025]
See also [Sauerwein et al. 2023]

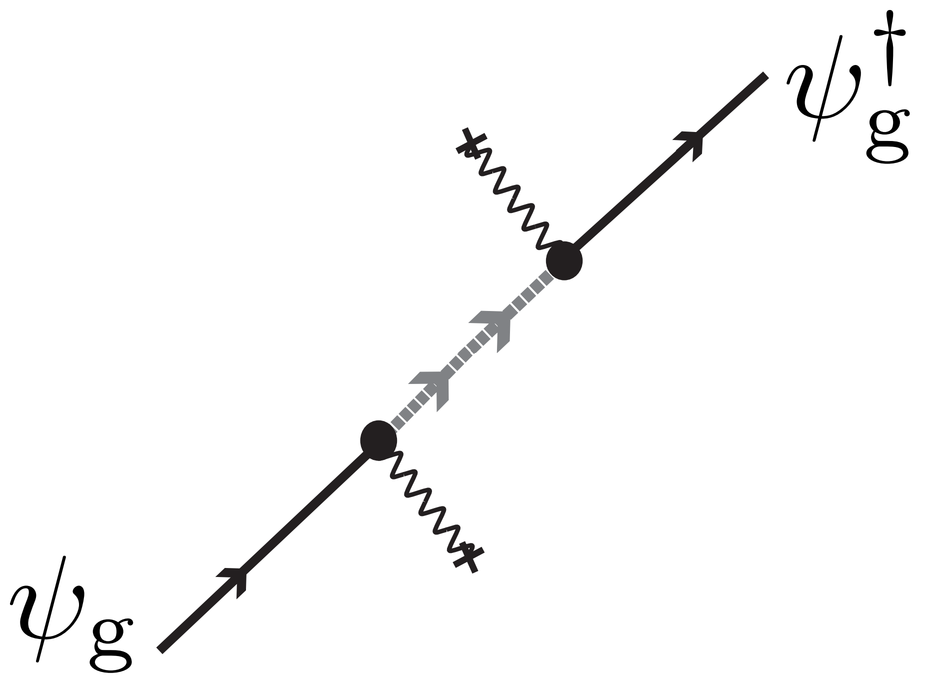



Interaction term: $$\frac{\Omega_d^2\Omega_c^2}{4\Delta_{cd}\Delta_{ad}^2}\int d^2r\, d^2r' \frac{\Delta_{ad}^2}{\Delta_{ad}(r)\Delta_{ad}(r')}\psi_g^\dagger(r)\psi_g(r)\psi_g^\dagger(r')\psi_g(r')$$

# Diagrams

[Uhrich et al. 2023]
[Baumgartner et al. 2024 & 2025]
See also [Sauerwein et al. 2023]

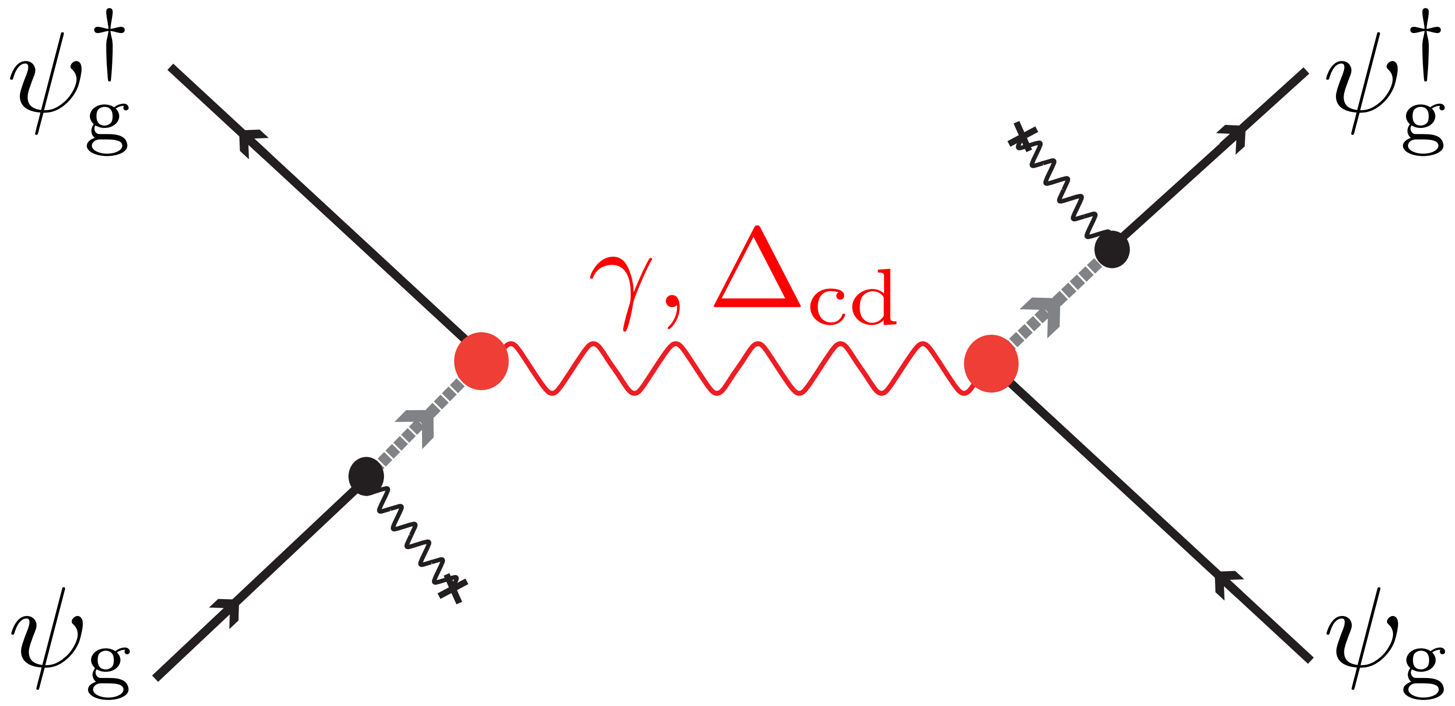
## Two body




$$\frac{\Omega_d^2}{\Delta_{ad}} \int d^2 r \, \frac{\Delta_{ad}}{\Delta_{ad}(r)} \, \psi_g^\dagger(r) \psi_g(r)$$

SYK₂ like term that has to be compensated

## Four body




$$\frac{\Omega_d^2 \Omega_c^2}{4 \Delta_{cd} \Delta_{ad}^2} \int d^2 r \, d^2 r' \, \frac{\Delta_{ad}^2}{\Delta_{ad}(r) \Delta_{ad}(r')} \, \psi_g^\dagger(r) \psi_g(r) \psi_g^\dagger(r') \psi_g(r')$$

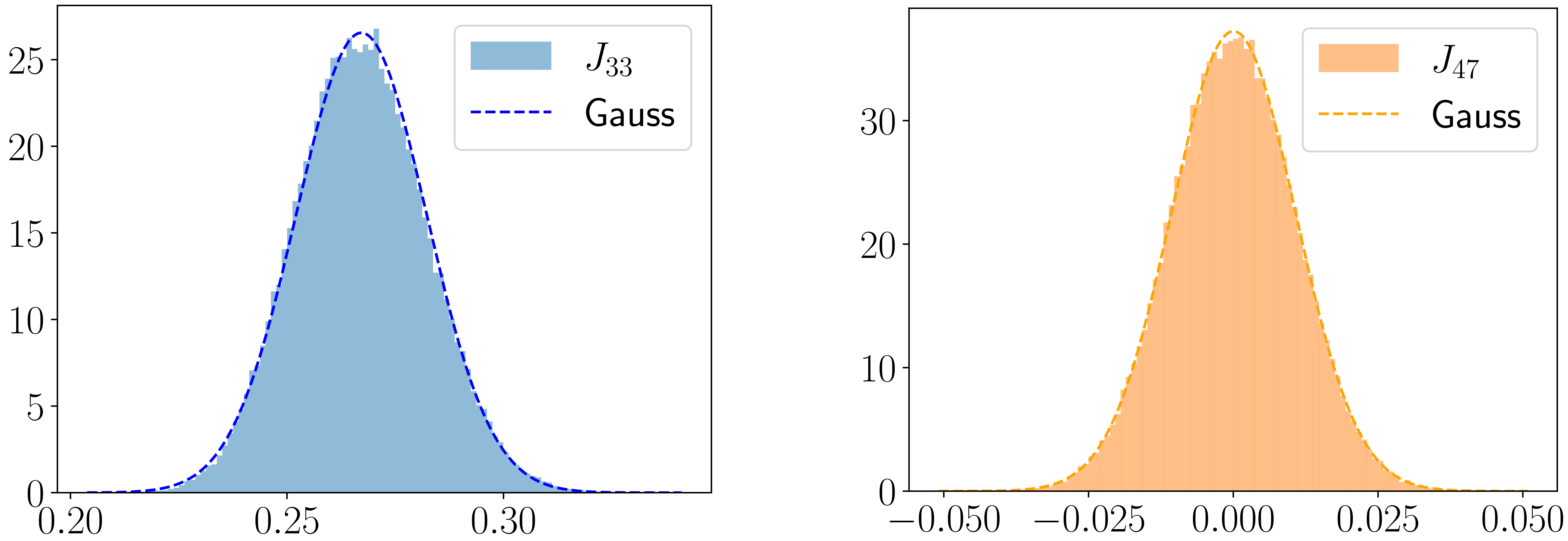
$$H_{\text{eff}} = \frac{\Omega_d^2 \Omega_c^2}{\Delta_{cd} \Delta_{ad}^2} \sum_{i_1, i_2; k_1, k_2} J_{i_1 k_1} J_{i_2 k_2} c_{i_1}^\dagger c_{k_1} c_{i_2}^\dagger c_{k_2}$$

‘low-rank’ version of SYK

# Experimental couplings

[Baumgartner et al. 2024 & 2025]

$$J_{ij} = \frac{1}{2}\int \mathrm{d}^2 r \, \frac{\phi_i^*(r)\,\phi_j(r)}{1+S(r)} \longrightarrow \overline{J_{ij}} = \overline{\Big(2\big(1+S(r)\big)\Big)^{-1}}\,\delta_{ij}$$



# Number of interacting sites $N$

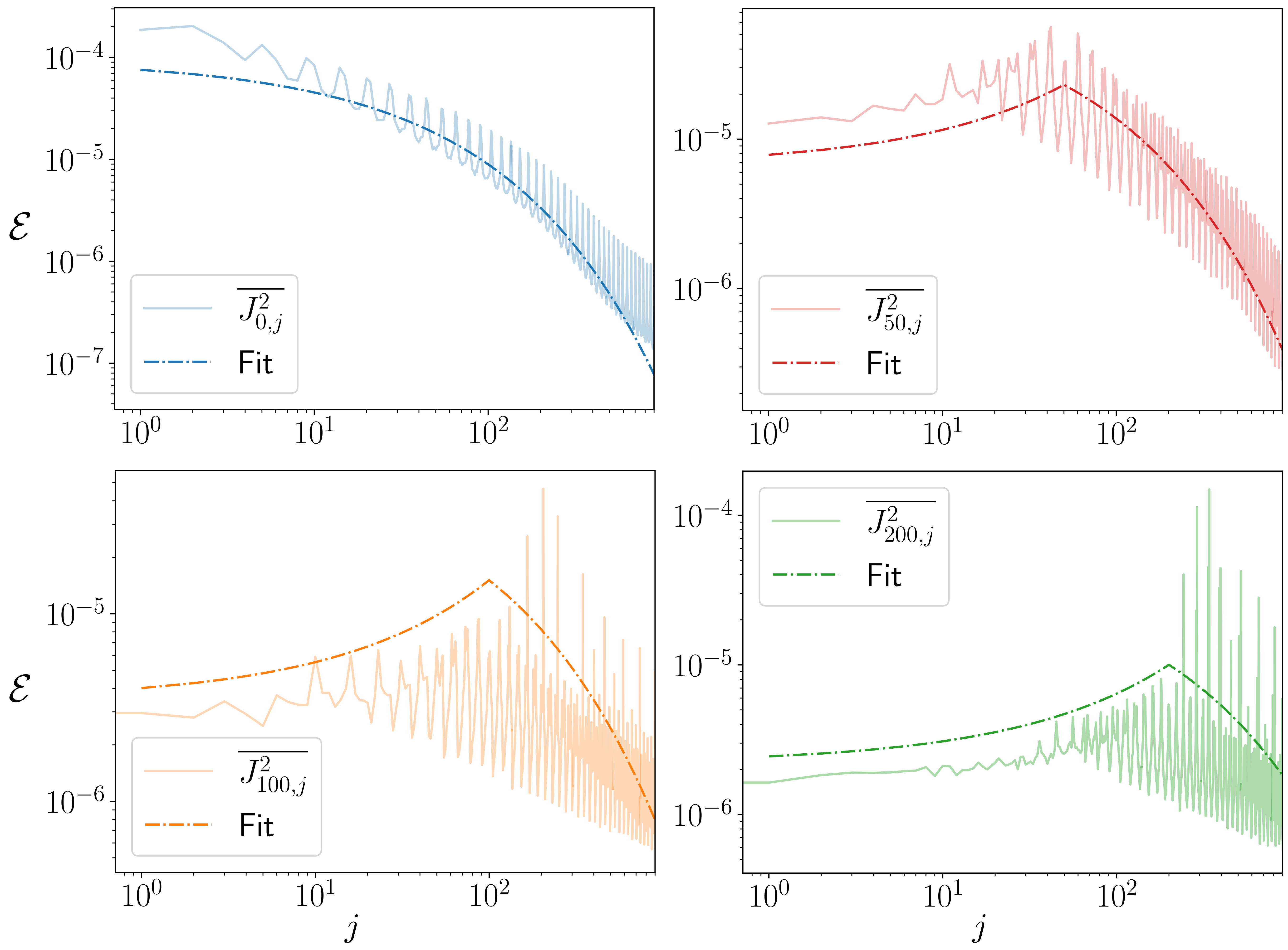
[Baumgartner et al. 2025]

Fix $i$ and scan the variance of $J_{ij}$ for different values of $j$.

Empirical ansatz:

$$\overline{J_{ij}^2} \sim \exp\left(\left|\frac{\sqrt{i}-\sqrt{j}}{\sqrt{N_{\text{eff}}}}\right|\right)$$

| $i$ | $r = 6$ | $r = 15$ |
|---|---|---|
| 0 | $N_0 \approx 20$ | $N_0 \approx 120$ |
| 50 | $N_{50} \approx 31$ | $N_{50} \approx 129$ |
| 100 | $N_{100} \approx 46$ | $N_{100} \approx 151$ |
| 200 | $N_{200} \approx 86$ | $N_{200} \approx 170$ |



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[Baumgartner et al. 2025]

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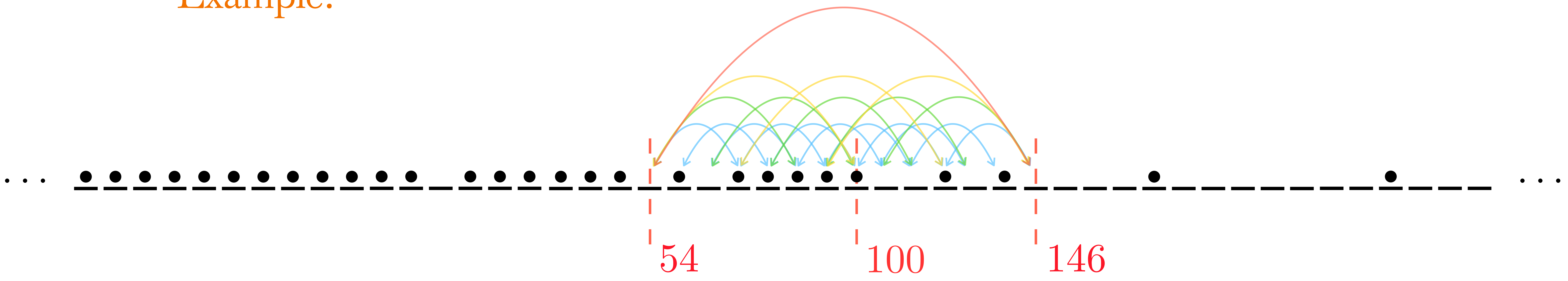
| 100 | $N_{100} \approx 46$ | $N_{100} \approx 151$ |
| 200 | $N_{200} \approx 86$ | $N_{200} \approx 170$ |

Main takeaway: the number of SYK interacting sites $N$ is *not* equal to the number of atoms $N_{at}$ in the cavity.

It is directly related to the speckle pattern.

Example:



# Trotterization to densify disorder

$J_{i_1 i_2; k_1 k_2}$

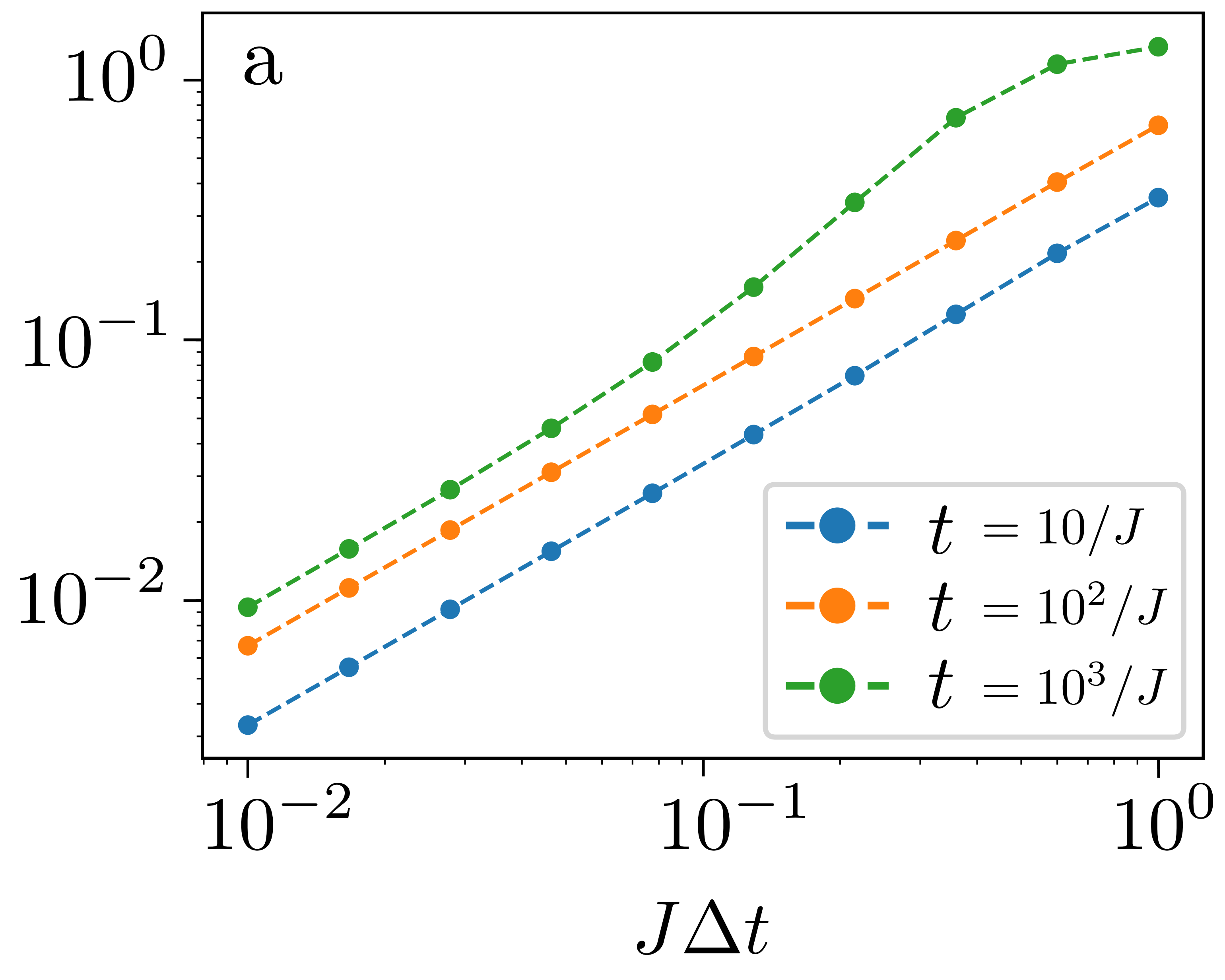
Trotterization: $$H = \sum_{n=1}^{R} H_{\text{eff}}^{n} = \mathcal{E} \sum_{i_1 i_2; k_1 k_2} \left( \sum_{n=1}^{R} J_{i_1 k_1}^{n} J_{i_2 k_2}^{n} \right) c_{i_1}^{\dagger} c_{k_1} c_{i_2}^{\dagger} c_{k_2}$$

[Kim et al. 2019] : $R = \gamma N$ gives $\text{SYK}_2$ to $\text{SYK}_4$ interpolation

$$U_{\text{eff}}(T) \equiv e^{-iH_{\text{eff}} t} = \left( \prod_{\alpha=1}^{R} e^{-iH_\alpha t/n} \right)^n + \frac{t^2}{2n} \sum_{\alpha<\beta}^{R} [H_\alpha, H_\beta] + \dots$$

$$\overline{\left\| \sum_{\alpha<\beta} [H_\alpha, H_\beta] \right\|^2} \lesssim 2 \times 10^2 \frac{J^4 R^2}{N^2} + \mathcal{O}(R^2/N^3)$$

$\Delta U$

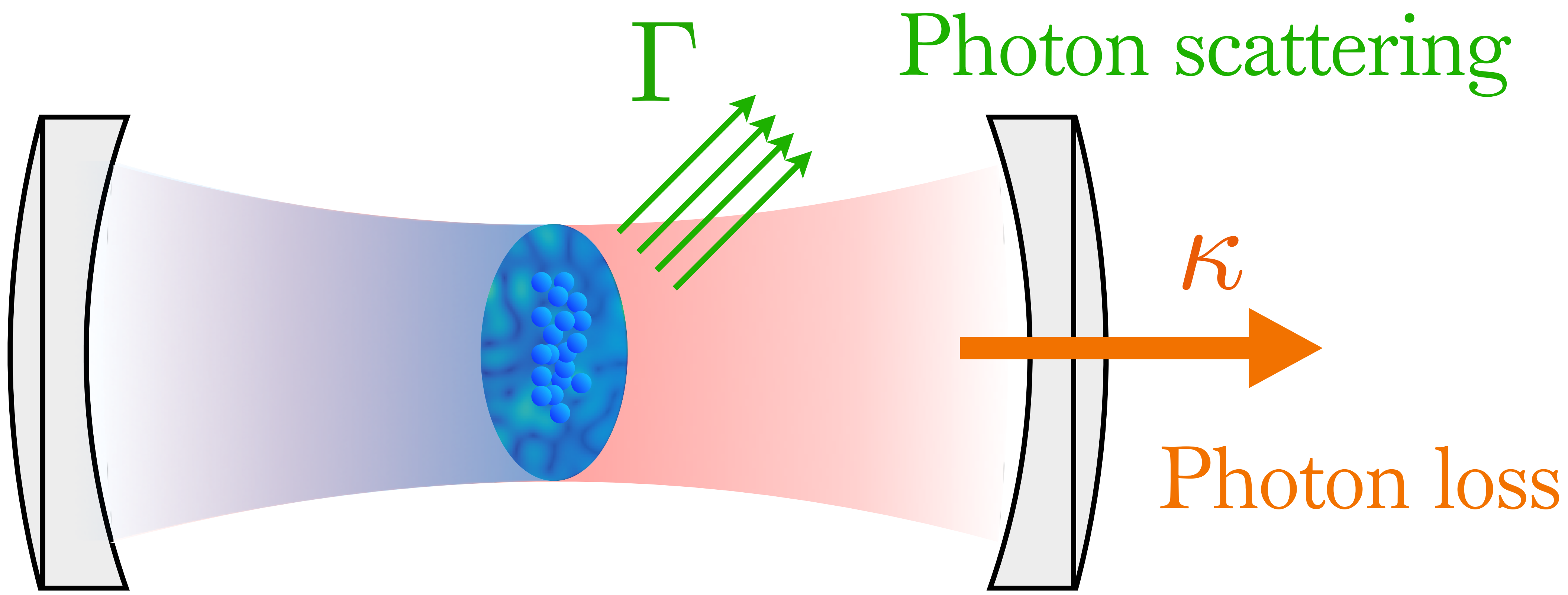



[Baumgartner et al. 2024 & 2025]

Spars*er* but not too sparse!

# Dissipation

[Baumgartner et al. 2024 & 2025]



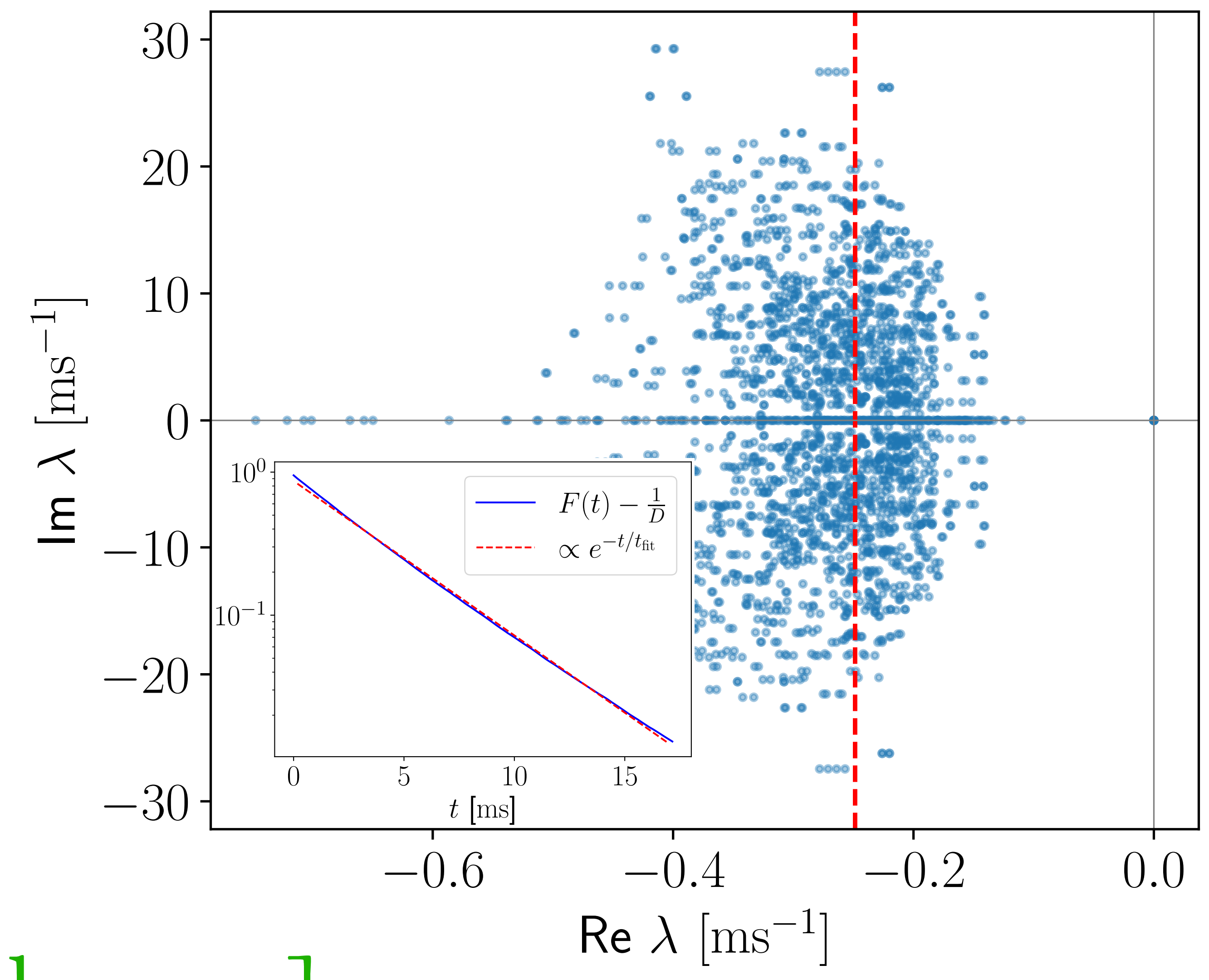

$$L_\Gamma(r) = \frac{\sqrt{\Gamma}\,\Omega_{\rm d}\, g_{\rm d}(r)}{\Delta_{\rm ad}(r) + i\Gamma/2}\,\psi_{\rm g}^\dagger(r)\psi_{\rm g}(r)$$

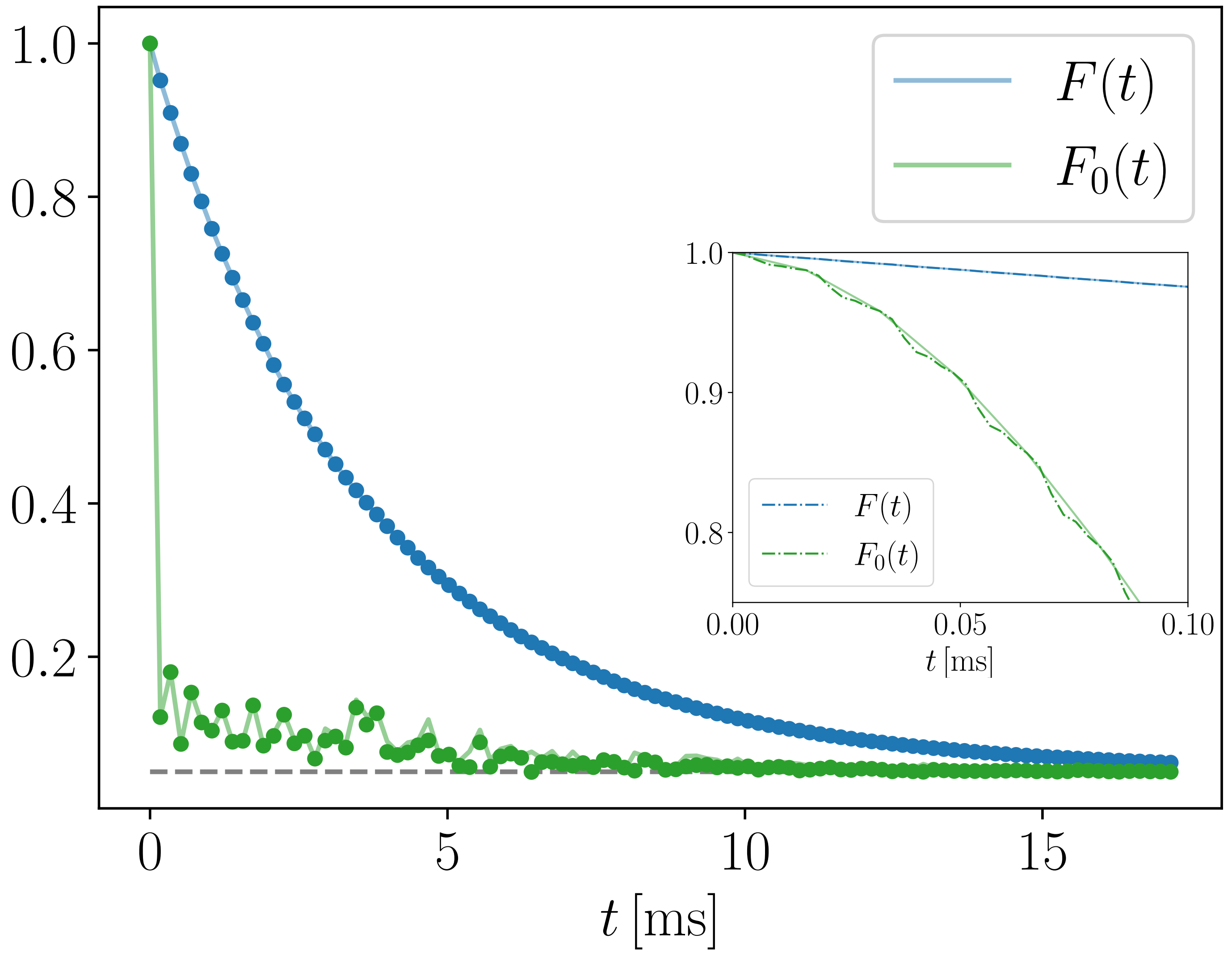
$$L_\kappa = \frac{\sqrt{\kappa}\,\Omega_{\rm d}\Omega_{\rm c}}{2(\Delta_{\rm cd} - i\kappa/2)} \int {\rm d}^2 r\, \frac{g_{\rm d}(r) g_0(r)}{\Delta_{\rm ad}(r) + i\Gamma/2}\,\psi_{\rm g}^\dagger(r)\psi_{\rm g}(r)$$

Survival fidelity $F_0(t)$ vs coherent fidelity $F(t)$ for photon loss.

Recent activity on: how much do quadratic Lindblad jump operators affect chaotic features of SYK?

[Kulkarni et al. 2021][Sá et al. 2021]
[Kawabata et al. 2022][A.M. García-García et al. 2024] …



# Summary

Holographic systems are interesting states of matter whose exploration will deepen our understanding of many areas of physics.

Realizing the SYK model is a first step towards scalable quantum simulations of more complicated holographic systems.

In cQED platforms, we showed that it is possible to have

- *Tunable* (large) number of interacting degrees of freedom
- Programmable *control over couplings*, at least to generate disorder
- Generate the *all-to-all connectivity* by cavity photons

# Future directions

Many near-term goals and forthcoming challenges:

- *Upgrade the cavity with additional speckle* (@ LQG-EPFL by Brantut's group)
- *Cool down* the system to reach strong coupling
- *Find observables* that can be realistically measured in the lab and have a holographic interpretation. Some examples to explore: [García-Álvarez et al. 2016]
  1. Correlation functions: 2pt universal, 4pt related to OTOCS [Swingle et al. 2016]
  2. SYK thermodynamics? [Vermersch et al. 2019]
  3. Statistical approaches, as *e.g.* [Zhou et al. 2025 & 2025] ?
- Address *interplay between dissipative dynamics and quantum chaos*. Is there any lessons to be learned from the bulk?

# Thank you!

# Backup slides

# Low dimensional gravity

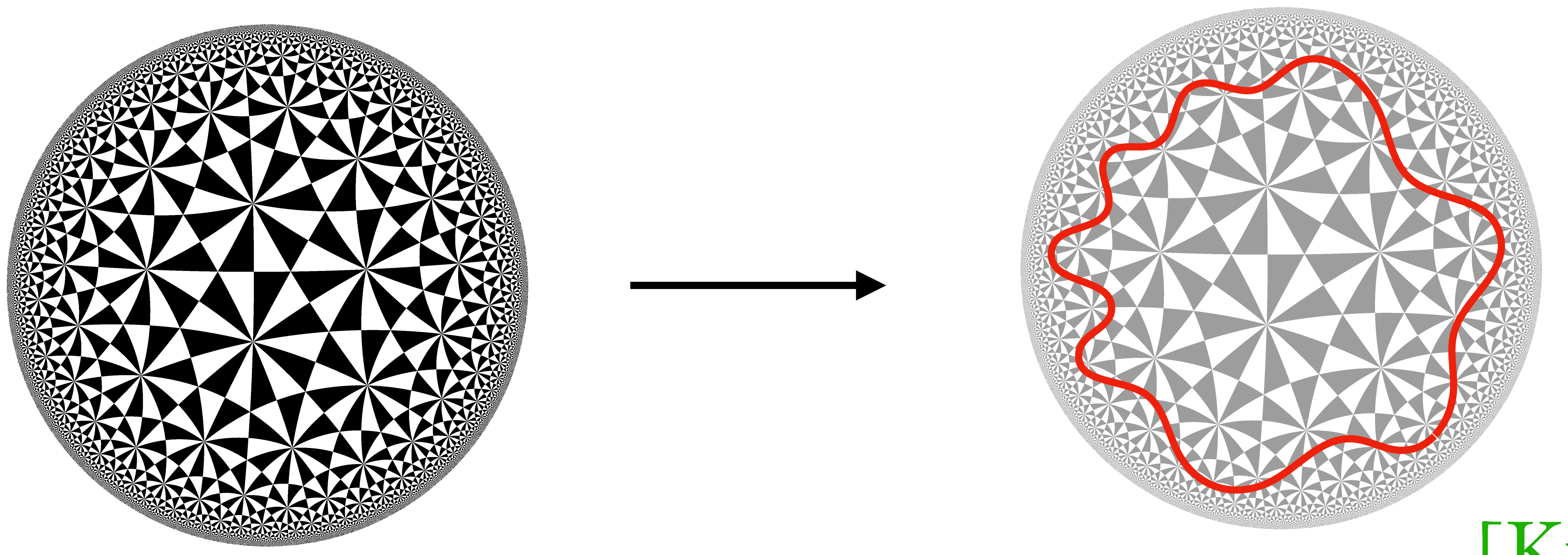
n. of gravitons: $d(d-3)/2$

$d = 4$ 2 gravitons
$d = 3$ topological
$d = 2$ -1 dof

By $AdS_{d+1}/CFT_d$ the gravitational theory is two dimensional

JT gravity: $$I = -\chi S_0 - \frac{1}{16\pi G}\int_{\mathcal{M}} \mathrm{d}^2x\,\sqrt{g}\,\Phi(R+2) - \frac{1}{8\pi G}\oint_{\partial\mathcal{M}} \mathrm{d}\theta\,\sqrt{h}\,\Phi(K-1)$$

Adding the dilaton $\Phi$, the theory is topological. On closed manifolds it is actually trivial. To add dynamics we need a boundary.



Schwarzian action weighs boundary reparametrizations. It appears also in SYK as the action for the pseudo-Goldstone bosons for broken conformal symmetry

[Kitaev 2015] [Maldacena, Stanford 2015]

# How many $R$'s?

$$R = \gamma N^{\alpha}$$

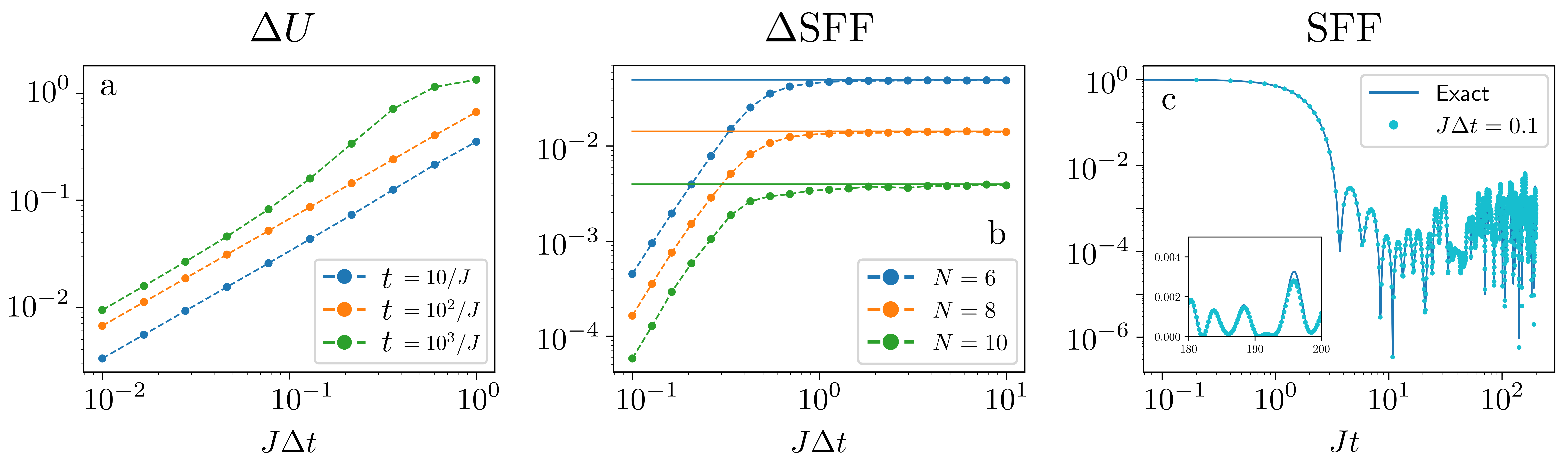
$\alpha < 1 \longrightarrow \begin{cases} T > T_c & \text{Free fermions + no condensation} \\ T \leq T_c & \text{SYK}_p \text{ + boson condensation} \end{cases}$

$\alpha = 1 \longrightarrow$ Interpolation $\text{SYK}_p$ to $\text{SYK}_{2p}$ + no condensation

$\alpha > 1 \longrightarrow$ $\text{SYK}_{2p}$ + no condensation

[Esterlis, Schmalian 2019]
[Kim, Cao, Altman 2019]

# Bounds on trotterization error




$$U_{\text{eff}}(T) \equiv e^{-iH_{\text{eff}}t} = \left(\prod_{\alpha=1}^{R} e^{-iH_\alpha t/n}\right)^n + \frac{t^2}{2n}\sum_{\alpha<\beta}^{R}[H_\alpha, H_\beta] + \dots$$

$$\overline{\left\|\sum_{\alpha<\beta}[H_\alpha, H_\beta]\right\|^2} \lesssim 2\times 10^2\,\frac{J^4R^2}{N^2} + \mathcal{O}(R^2/N^3)$$

# Converging couplings

$$P(\mathcal{Y})\,\mathrm{d}\mathcal{Y} = \frac{\sqrt{R}}{\sqrt{\pi}\,\sigma^2\,\Gamma(R/2)} \left(\frac{\sqrt{R}\,|\mathcal{Y}|}{2\sigma^2}\right)^{\frac{R-1}{2}} K_{\frac{1-R}{2}}\left(\frac{\sqrt{R}\,|\mathcal{Y}|}{\sigma^2}\right)\mathrm{d}\mathcal{Y}$$

$$= \frac{e^{-\frac{\mathcal{Y}^2}{2\sigma^4}}}{\sqrt{2\pi\sigma^4}} + \frac{1}{R}\left(\frac{3}{4} - \frac{3\mathcal{Y}^2}{2\sigma^4} + \frac{\mathcal{Y}^4}{4\sigma^8}\right)\frac{e^{-\frac{\mathcal{Y}^2}{2\sigma^4}}}{\sqrt{2\pi\sigma^4}}$$

$$+\frac{1}{R^2}\left(\frac{25}{32} - \frac{45\mathcal{Y}^2}{8\sigma^4} + \frac{65\mathcal{Y}^4}{16\sigma^8} - \frac{17\mathcal{Y}^6}{24\sigma^{12}} + \frac{\mathcal{Y}^8}{32\sigma^{16}}\right)\frac{e^{-\frac{\mathcal{Y}^2}{2\sigma^4}}}{\sqrt{2\pi\sigma^4}} + \mathcal{O}(R^{-3})$$

# Converging couplings

$$Y = X_1 X_2 \longrightarrow P(Y)\,\mathrm{d}Y = \frac{1}{\pi\sigma^2}\,K_0\left(\frac{|Y|}{\sigma^2}\right)\,\mathrm{d}Y$$

$$\mathcal{Y} = \frac{1}{\sqrt{R}}\sum_{\alpha=1}^{R} Y_\alpha$$

$$P(\mathcal{Y})\,\mathrm{d}\mathcal{Y} = \frac{\sqrt{R}}{\sqrt{\pi}\,\sigma^2\,\Gamma(R/2)}\left(\frac{\sqrt{R}\,|\mathcal{Y}|}{2\sigma^2}\right)^{\frac{R-1}{2}} K_{\frac{1-R}{2}}\left(\frac{\sqrt{R}\,|\mathcal{Y}|}{\sigma^2}\right)\,\mathrm{d}\mathcal{Y}$$

# Scaling dimension

[Esterlis, Schmalian 2019]
[Kim, Cao, Altman 2019]

$$R = \gamma N$$