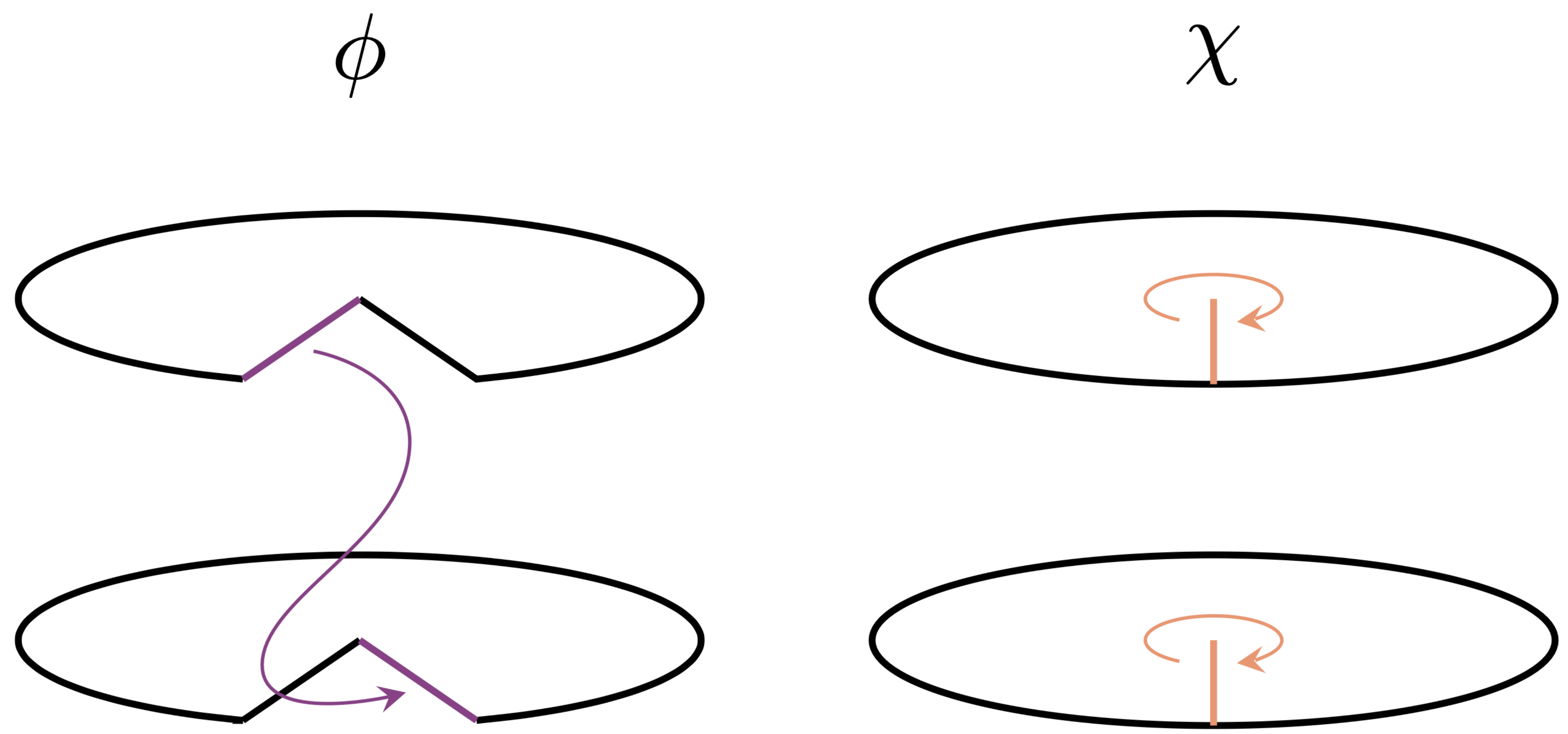
Black Holes as open quantum systems and unitary dynamics

Based on upcoming work with
J. Sonner



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A story 50 years long

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Can we still learn something from it?

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We want to study

$$\rho_{\text{sys}}(t) = \text{Tr}_{\text{env}}[\, \rho(t) \,]$$

Open Effective Field Theory

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$$I[\phi, \chi] = I_{\text{sys}}[\phi] + I_{\text{env}}[\chi] + \eta \int \mathrm{d}x \; \phi(x)\, \chi(x)$$

one obtains

$$\mathcal{F}[\phi] = Z_\chi \exp\left(\sum_{l=1}^{\infty} \frac{(-1)^l\, \eta^l}{l!} \int \mathrm{d}x_1 \ldots \mathrm{d}x_l\, \phi(x_1) \ldots \phi(x_l)\, G^c_\chi(x_1, \ldots, x_l) \right)$$

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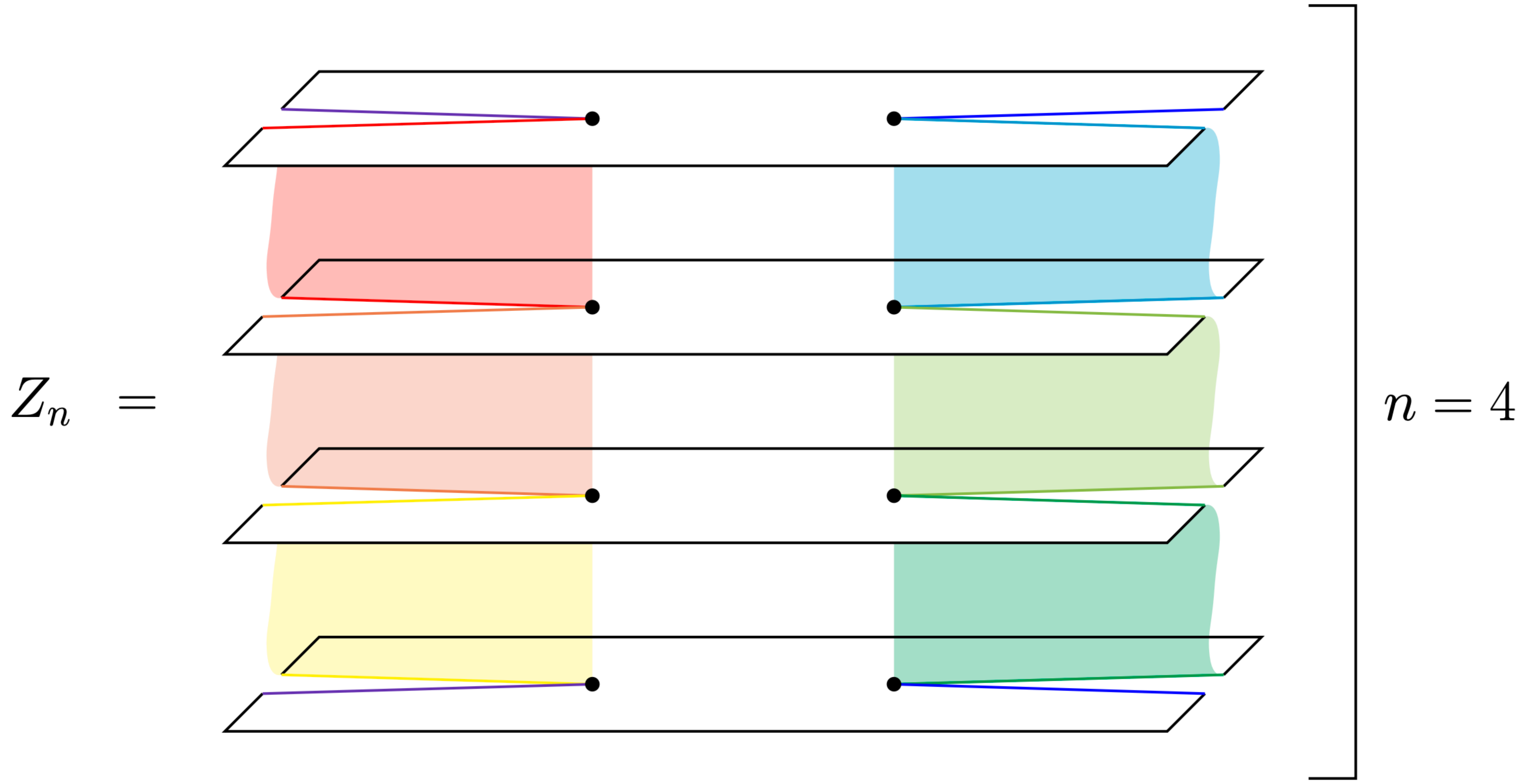

Entanglement Entropy for QFTs

Using the replica trick we can compute the EE for a free scalar in 2D

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Then if

$$I[\phi, \chi] = \int \mathrm{d}^2 x \left(\frac{1}{2}\, \partial_\mu \phi\, \partial^\mu \phi + \frac{1}{2}\, m^2\, \phi^2 + \frac{1}{2}\, \partial_\mu \chi\, \partial^\mu \chi + \frac{1}{2}\, m^2\, \chi^2 + \eta\, \phi\, \chi \right)$$

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the entropy is

$$S_{\phi,\chi} = \frac{1}{6}\log\left(\frac{1}{m_+\,\varepsilon}\right) + \frac{1}{6}\log\left(\frac{1}{m_-\,\varepsilon}\right) = \frac{1}{3}\log\left(\frac{1}{m\,\varepsilon}\right) + \frac{\eta^2}{12m^4} + \dots$$

Entanglement Entropy from Open EFT

Let's compute the EE from the Open EFT. The replicated one is

$$Z_n = Z_\chi^n \int [\mathcal{D}\phi] \exp\left[-\int_{\mathcal{M}^n} \mathrm{d}^2x \left(\frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + \frac{1}{2}\,m^2\,\phi^2\right) + \frac{\eta^2}{2}\sum_{i=1}^{n}\int_{\mathcal{M}_i} \mathrm{d}^2x_1\,\mathrm{d}^2x_2\,\phi(x_1)\,\phi(x_2)\,G_\chi(x_1,x_2)\right]$$

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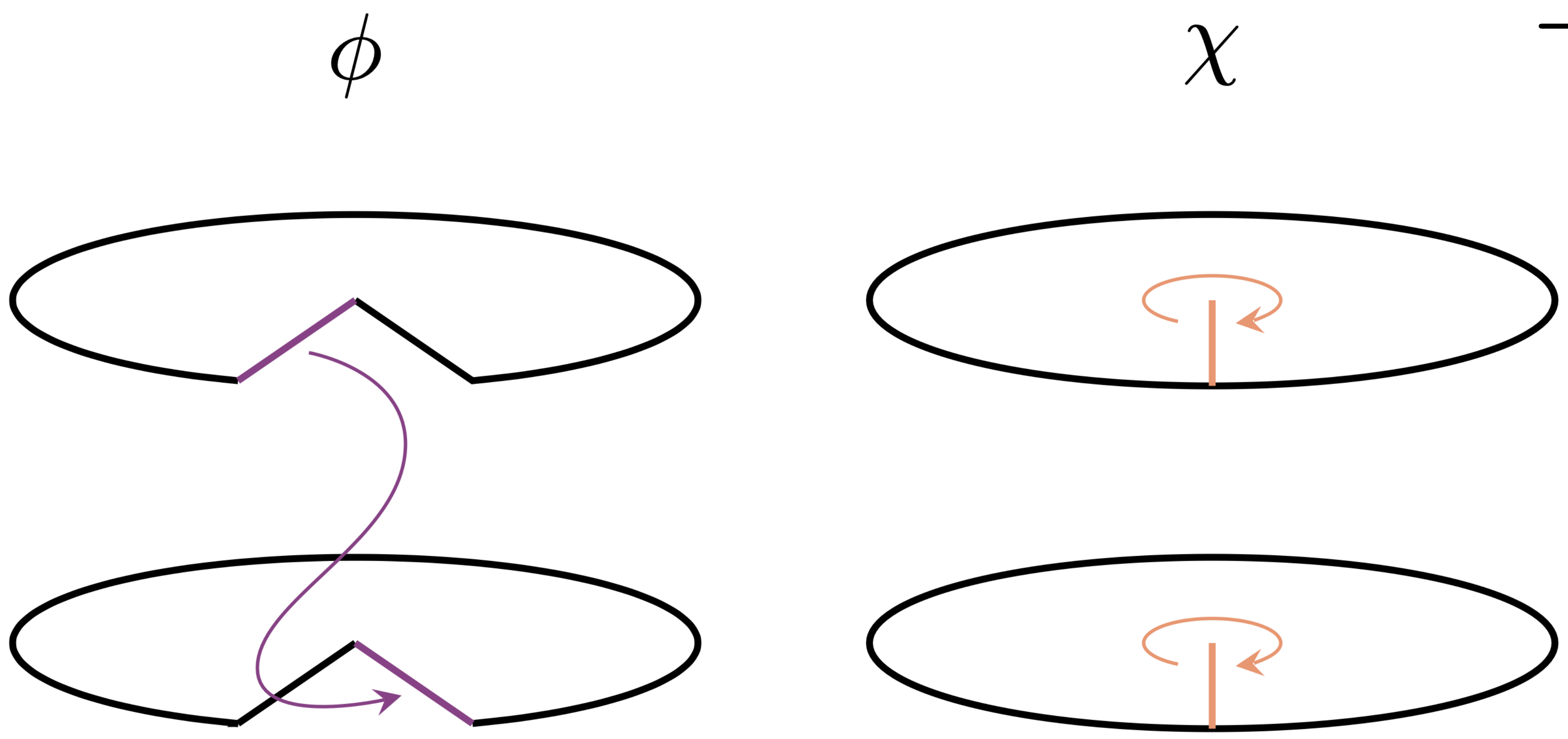

We are not coupling different replicas of the environment!

Entanglement Entropy from Open EFT

Computing the system entanglement entropy from this Open EFT we get

$$S_{\rm sys} = \frac{1}{6}\log\left(\frac{1}{m\,\varepsilon}\right) + \frac{\eta^2}{24m^4} + \ldots$$

The global one was

$$S_{\phi,\chi} = \frac{1}{3}\log\left(\frac{1}{m\,\varepsilon}\right) + \frac{\eta^2}{12m^4} + \ldots$$

What about gravity?

Gravity EFT and matter

We can use the same machinery for the EFT of gravity coupled to matter

$$I[g,\phi] = \frac{2}{\kappa^2}\int_{\mathcal{M}} \mathrm{d}^d x\,\sqrt{g}\left(R[g]-\Lambda\right) + \int_{\mathcal{M}\cup R} \mathrm{d}^d x\,\sqrt{g}\,\mathcal{L}_{\mathrm{mat}}[g,\phi]$$

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We expand in fluctuations from a saddle $g_{\mu\nu} = \hat{g}_{\mu\nu} + \kappa\, h_{\mu\nu}$ to obtain

$$I[h,\phi] = \frac{2}{\kappa^2}\int_{\mathcal{M}} \mathrm{d}^d x\,\sqrt{\hat{g}}\,\Big(R[\hat{g}]-\Lambda\Big) + \int_{\mathcal{M}} \mathrm{d}^d x\,\sqrt{\hat{g}}\,\mathcal{L}_{\mathrm{g}}[h]$$

$$+ \int_{\mathcal{M}\cup R} \mathrm{d}^d x\,\sqrt{\hat{g}}\,\mathcal{L}_{\mathrm{mat}}[\hat{g},\phi] + \kappa\int_{\mathcal{M}} \mathrm{d}^d x\,\sqrt{\hat{g}}\,h_{\mu\nu}\,T^{\mu\nu}[\hat{g},\phi] + \ldots$$

Gravity Influence Functional

We consider the matter (Hawking's radiation) to be the system of interest and gravity to be the environment.

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The Influence Functional then is

$$\mathcal{F}[\phi] = e^{-I_g[\hat{g}]}\, Z_h[\hat{g}]\, \exp\left(\frac{\kappa^2}{2}\int \mathrm{d}^d x\, \mathrm{d}^d y\, G_{\mu\nu,\rho\sigma}(x,y)\, T^{\mu\nu}(x)\, T^{\rho\sigma}(y) + \ldots\right)$$

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At leading order is a TT deformation of the matter theory!

Entanglement Entropy in the EFT approach

Computing the entanglement entropy for the matter theory coupled to gravity applying the EFT approach developed above we get

$$S_{\text{sys}} = S_\phi[\hat{g}] + \mathcal{O}(\kappa^2)$$

This is Hawking's result and it's not consistent with unitarity!

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However, we do know how to get a result consistent with unitarity, which is

$$S_{\text{sys}} = \min_I \left\{ \frac{8\pi A(\partial I)}{\kappa^2} + S_{\text{bulk fields}}(I \cup R) \right\}$$

Replicated Influence Functional for gravity

To obtain this result, we propose the following prescription to compute the replicated open effective theory.

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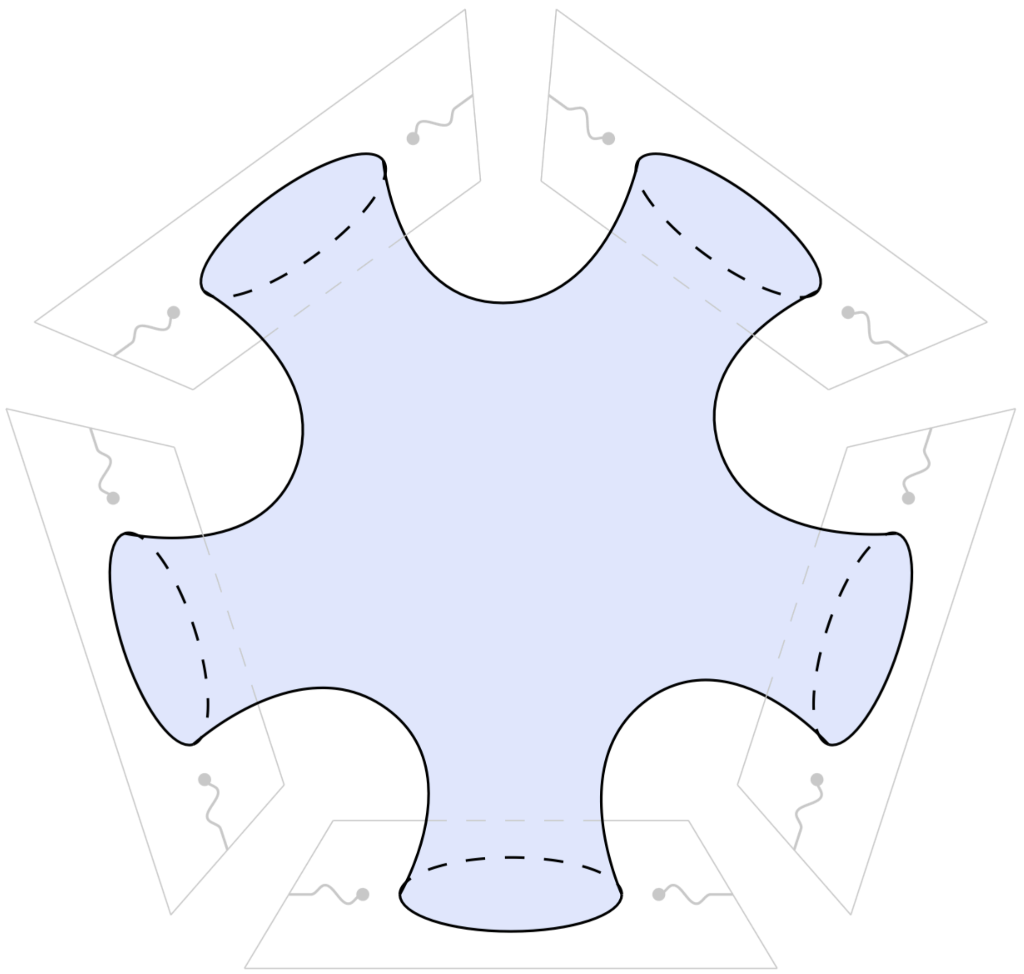

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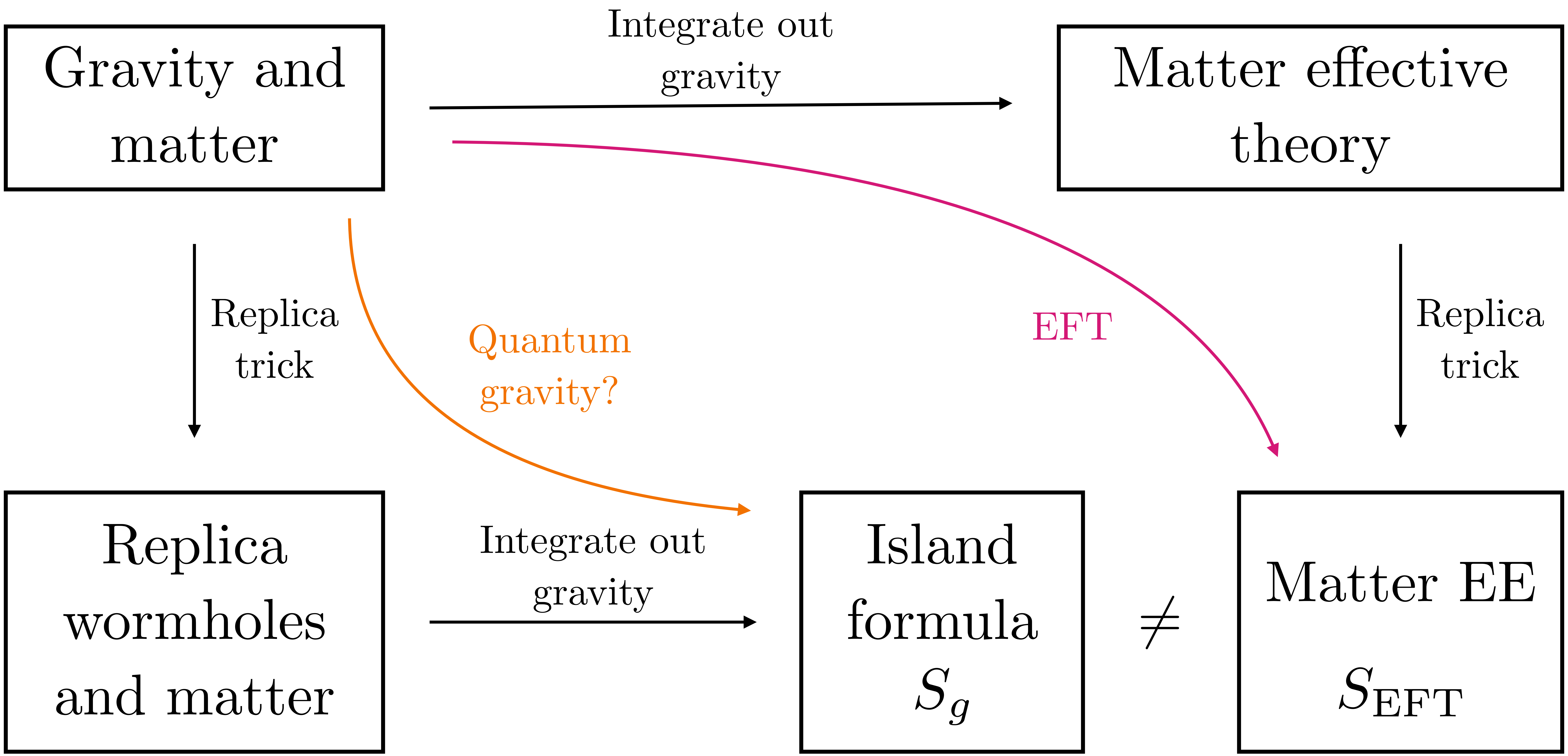
Following this prescription and computing the entanglement entropy we get the island formula.

A non-commutative diagram



Discussion

Results

- We computed the Open EFT for matter in a gravitational environment, which at leading order is a $T\bar{T}$-like deformation.
- We proposed a prescription to obtain the replicated partition function for systems in a gravitational environment

Future directions

- Can we learn something from this results? We believe so!
- Can we understand better this prescription from other pov's? Tensor networks?

Thank you!