Integrable Models and Supersymmetry Breaking in String Theory

Based on 2102.06184 in collaboration with A. Sagnotti

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Supersymmetry

Supersymmetry

Supersymmetry is a symmetry that exchanges bosonic and fermionic dof. We can define a spinorial supersymmetric charge Q such that, in flat space

$$Q|B,m\rangle \longrightarrow |F,m\rangle$$
$$Q|F,m\rangle \longrightarrow |B,m\rangle$$

The algebra of supersymmetric charges is

$$\{Q_\alpha, Q_\beta\} = 0 \qquad [Q_\alpha, P_\mu] = 0$$
$$[Q_\alpha, M_{\mu\nu}] = i(\sigma_{\mu\nu})_\alpha{}^\beta Q_\beta \qquad \{Q_\alpha, \overline{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$$

Gauging supersymmetry

Supersymmetry is closely related to gravity. In any theory of quantum gravity every symmetry must be gauged, since it's believed that global symmetries aren't allowed.

If we want to gauge supersymmetry we need to take into account that the theory must be invariant under local translations *i.e.* diffeomorphisms.

A theory in which Supersymmetry is gauged is then called *Supergravity*. All supergravity theories must contain

- a spin-2 massless field e_μ^a ,
- $\mathcal{N}$ spin-3/2 fields ψ_α^μ , the *Gravitini*.

Example: minimal supergravity D=4 found by Ferrara, Freedman and van Nieuwenhuizen.

Recap

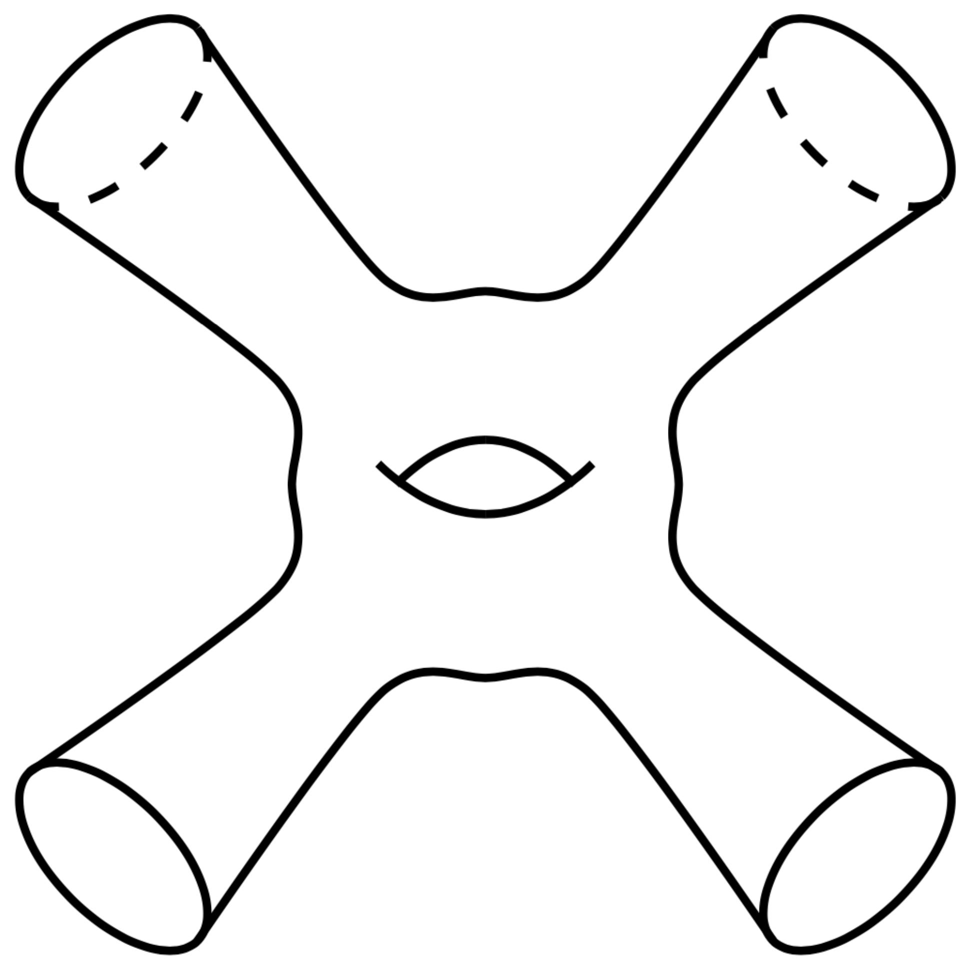
- Supersymmetry is a particular symmetry that connects bosonic and fermionic d.o.f.
- In a generic theory of quantum gravity, one expects all the symmetries to be gauged.
- All theories in which supersymmetry is gauged must comprise also gravity.
- Experiments tells us that supersymmetry, if at all realized in nature, must be spontaneously broken.

Question: Can we find a natural framework in which all these ingredients are realized?

Yes, String Theory!

String Theory

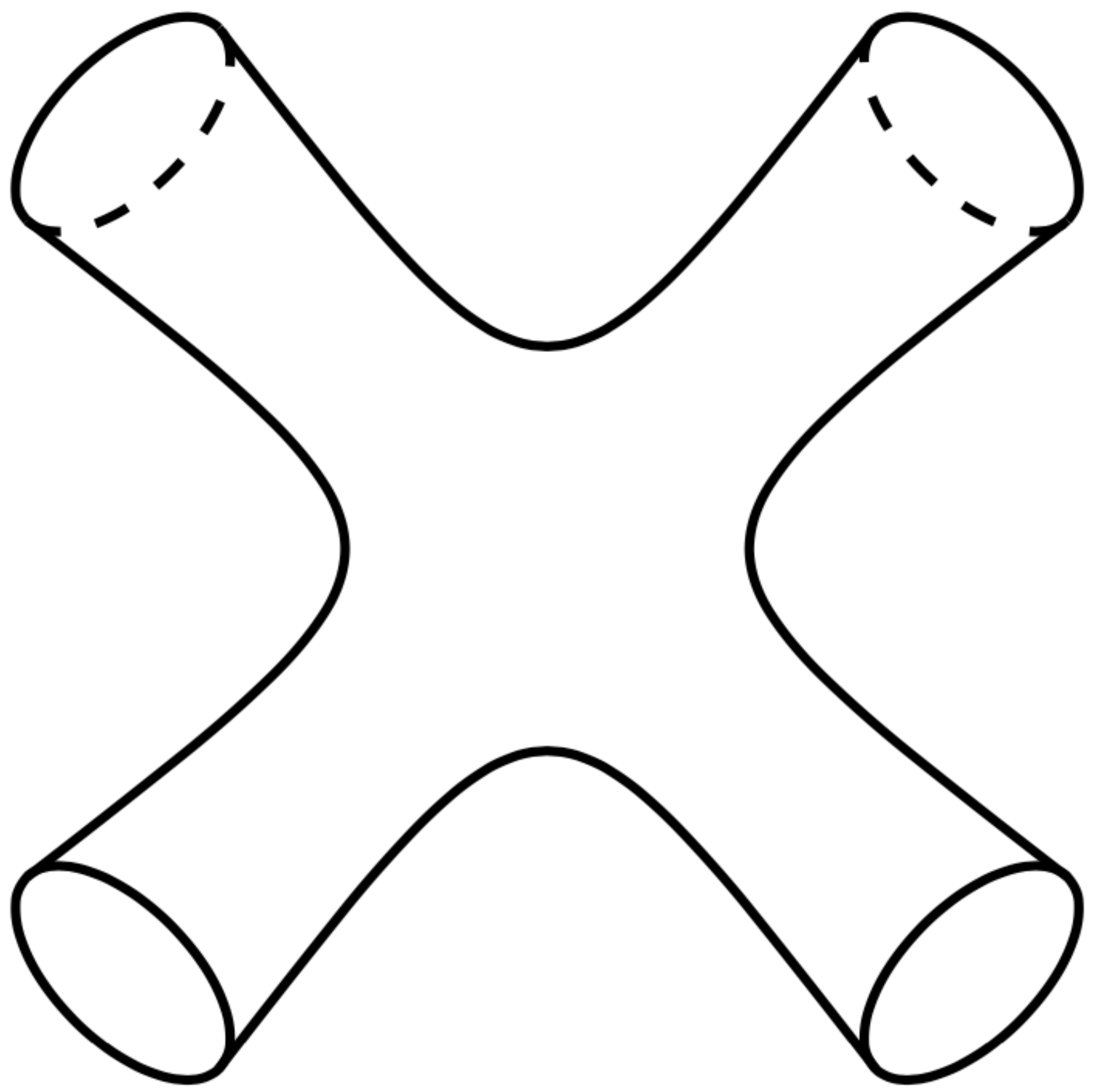
String Theory



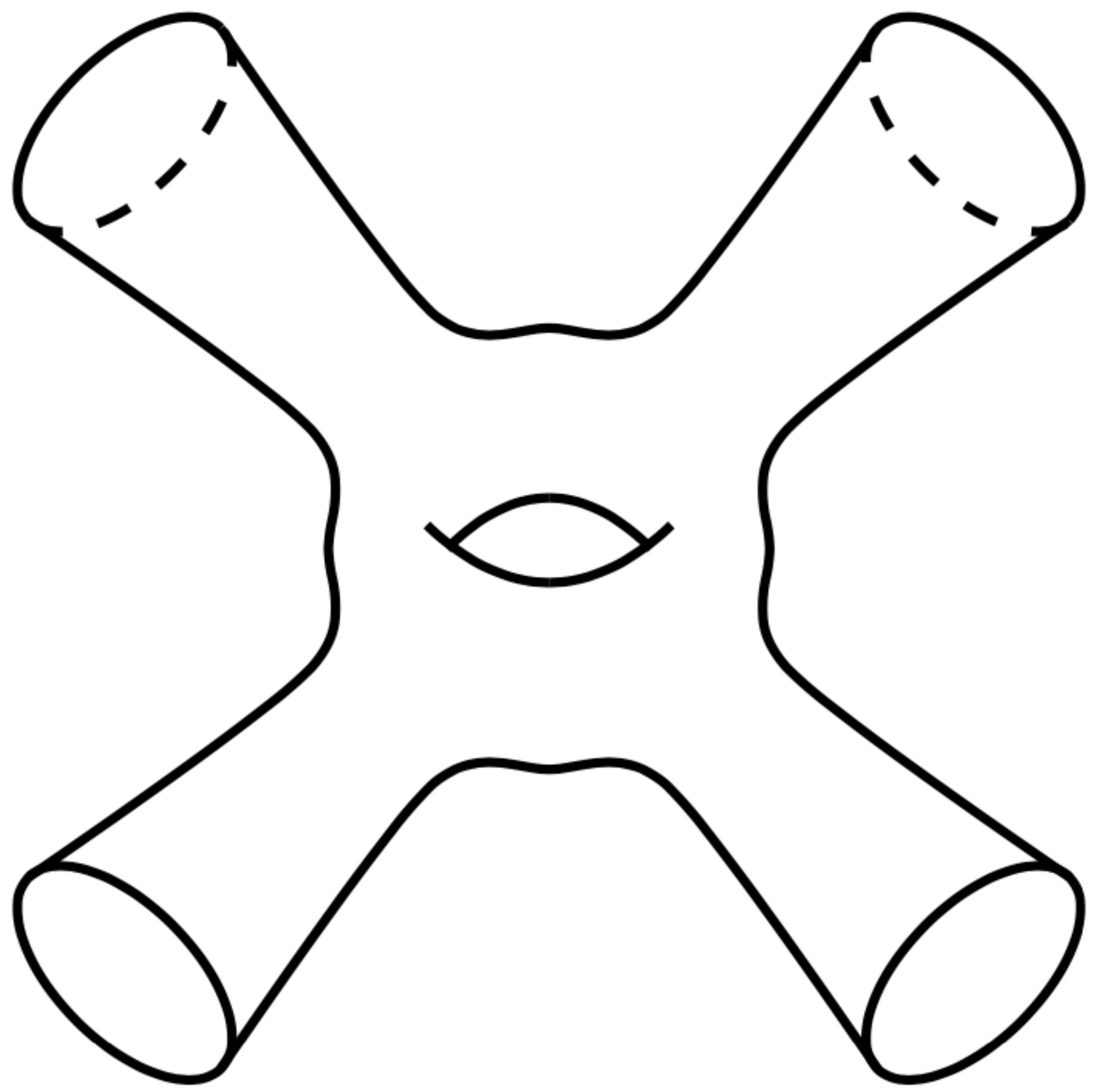
String Theory is a theory that maps a two dimensional worldsheet into a target space

$$S_P = -\frac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma \Big\{ \eta^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu - i\bar{\psi}^\mu \gamma^\alpha \partial_\alpha \psi_\mu \Big\}$$

Interactions



$$+$$



$$+ \quad \cdots$$

g_s^{-2} $\qquad$ g_s^{0}

$$g_s = e^{\phi}$$

Features

String Theory has a lot of features that makes it interesting, on top of which

- The target space must have D=10 for internal consistency,
- In the quantized theory gravity is unavoidable, as well as a dilaton field and a two form,

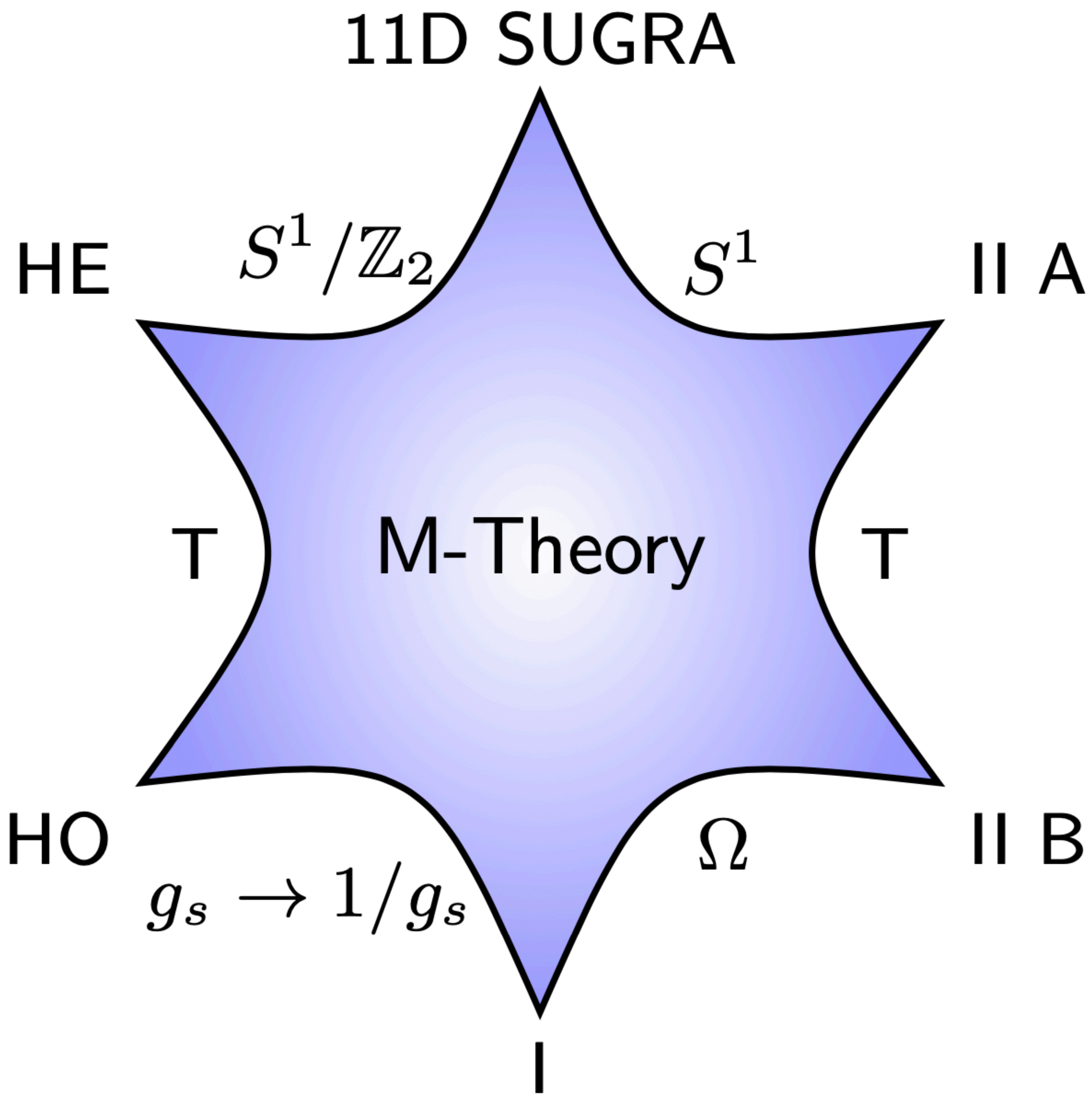
$$\left\{\phi, g_{\mu\nu}, B_{\mu\nu}\right\} \oplus \dots$$

- Supersymmetry appears very naturally, and it's gauged because it's a theory of quantum gravity,
- It contains extended objects (D-branes and orientifold planes) whose presence in the vacuum spontaneously break some (or all) of the supersymmetries.

We have good control on the theory only if the string coupling and the curvature of spacetime are small.

A Web of theories

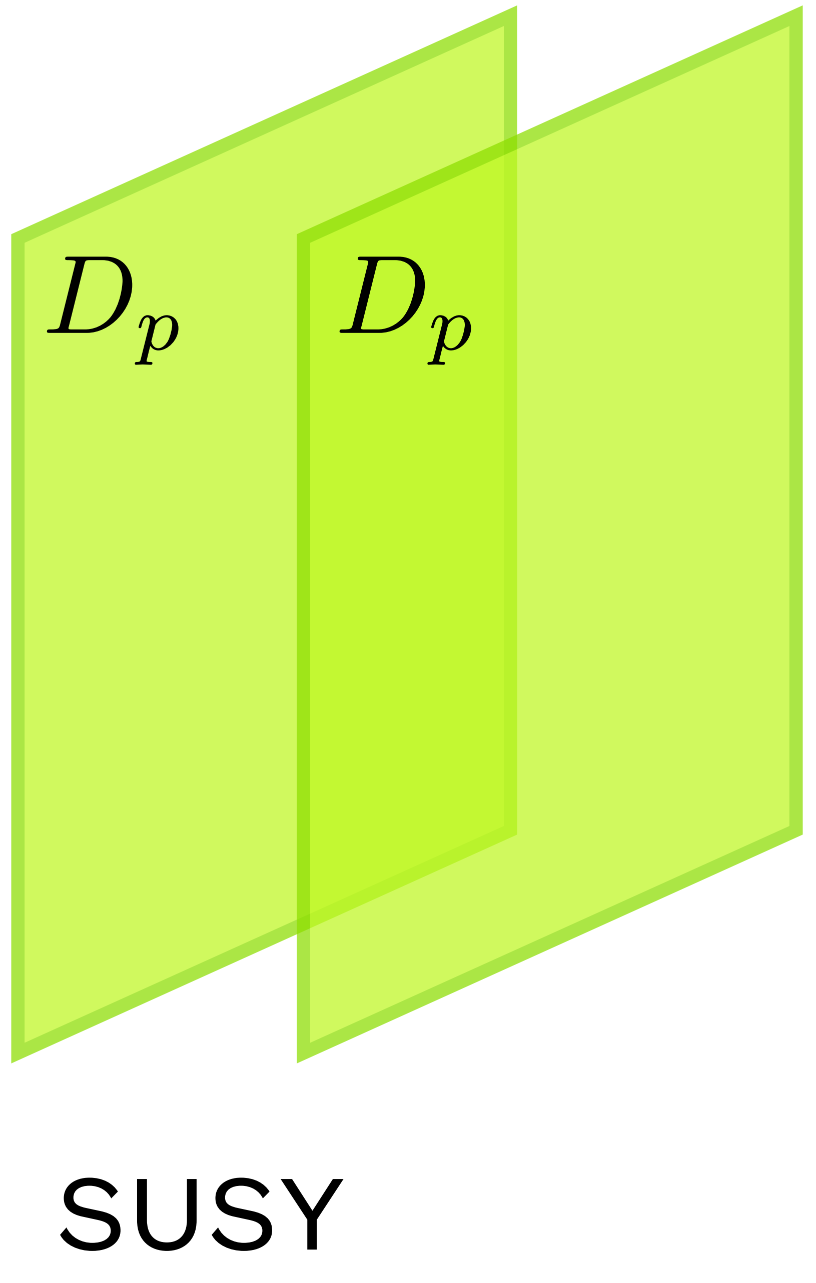
[Witten, 1995]

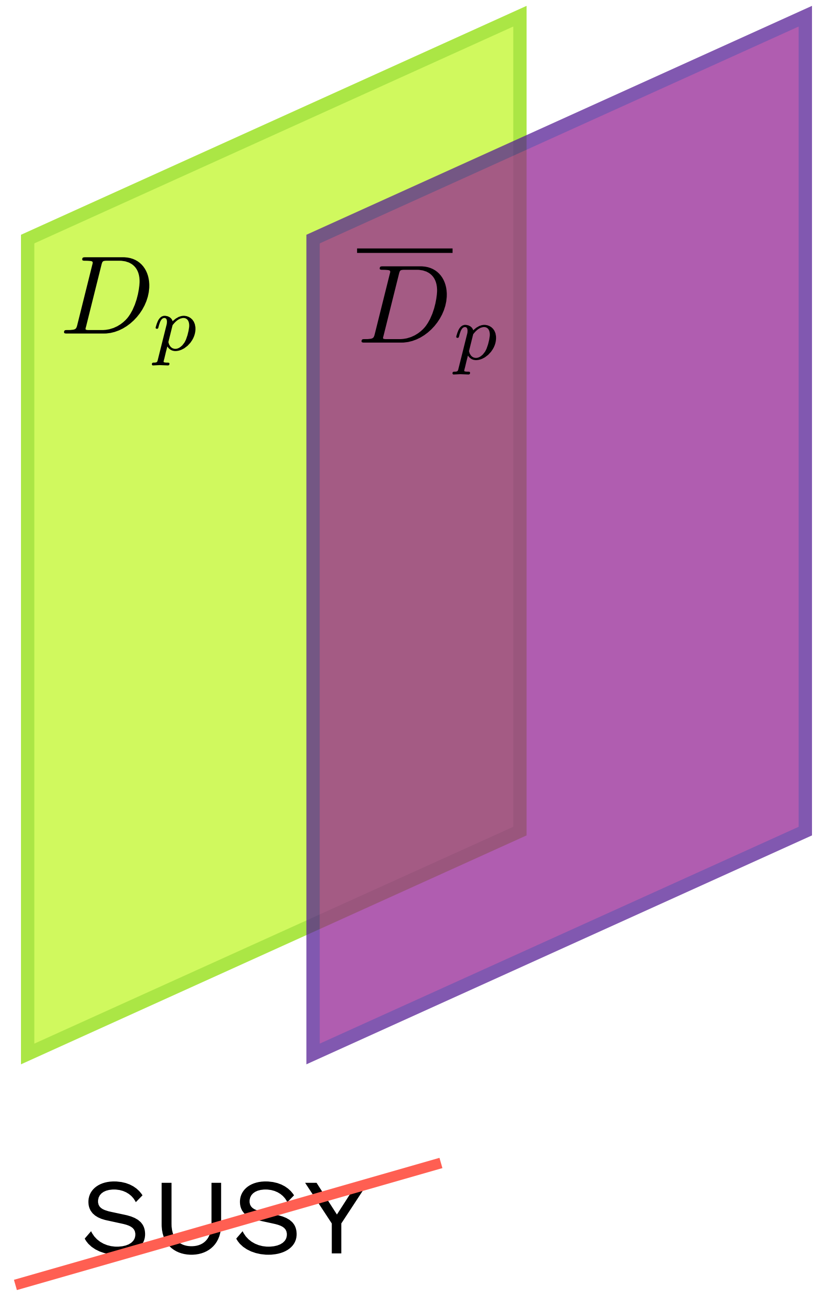


Breaking Supersymmetry

A way of breaking Supersymmetry

With branes and orientifolds we can break supersymmetries. However usually tachyons appear in the spectrum.





A way of breaking Supersymmetry

Usually, when we break all the supersymmetries with extended objects like branes and orientifolds, we get tachyons on the spectrum, which signals instabilities.

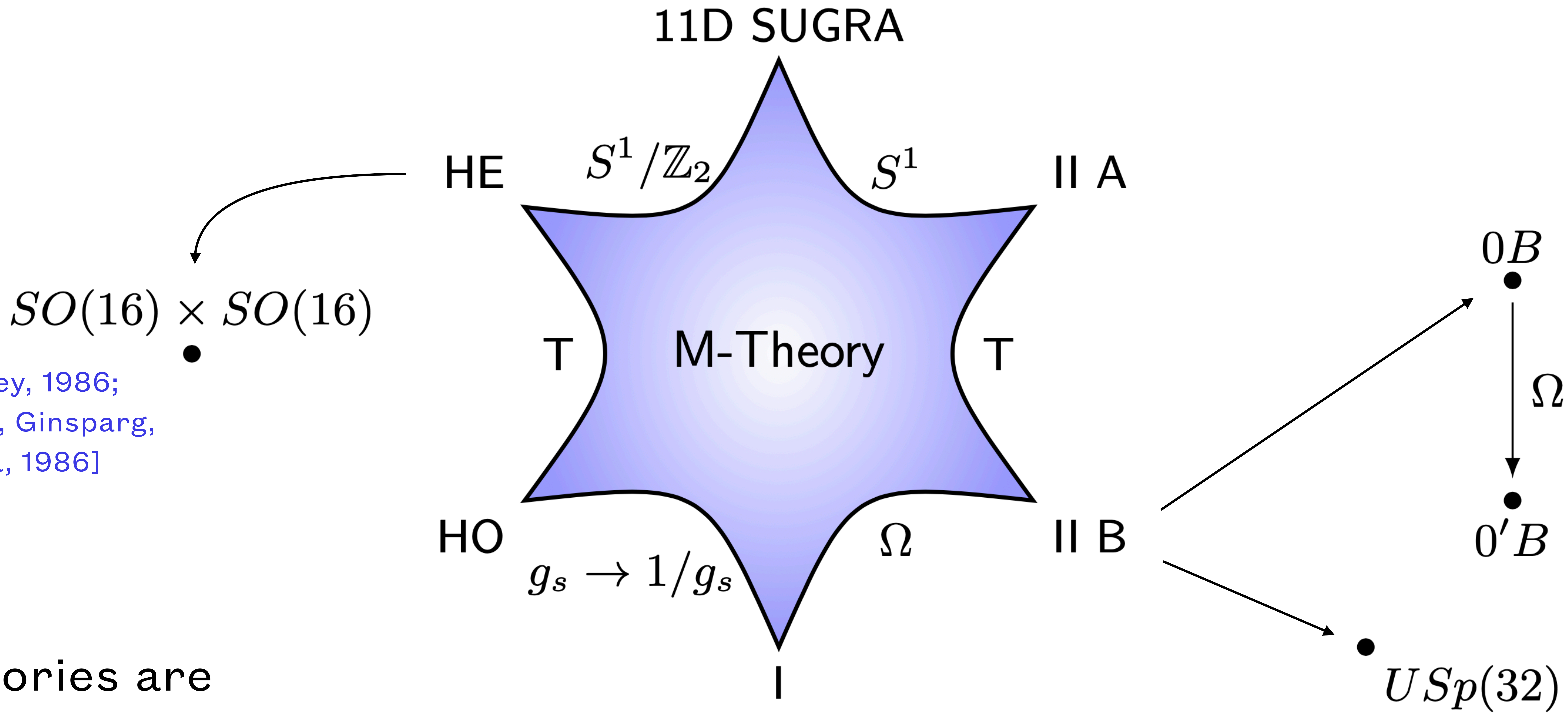
However, there are some models which do not display tachyonic modes. One particularly interesting is Brane Supersymmetry Breaking, which, in its simples realization, considers only 32 D9 branes and one O9 plane.

The fact that tachyons do not appear in the spectrum means that the model should be stable, at least perturbatively.

However, as we will see, the breaking of supersymmetry will have deep consequences for the vacuum of the theory.

[Sugimoto, 1999; Antoniadis, Dudas, Sagnotti, 1999]

A bigger web of theories



[Dixon, Harvey, 1986; Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1986]

[Sagnotti, 1995]

[Sugimoto, 1999; Antoniadis, Dudas, Sagnotti, 1999]

These theories are all **non-tachyonic**!

Low Energy EFT

The effective action is of the form

$$S = \int \mathrm{d}^{10}x\sqrt{-g}\left[R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \dots\right]$$

From the perturbative expansion we can write the potential as

$$V(\phi) = \sum_{n\in\mathbb{N}} c_n e^{(n+\frac{1}{2})\phi}$$

In our case (in Einstein frame)

$$V_{USp(32)}(\phi) = c_1 e^{\frac{3}{2}\phi} + \dots \qquad V_{0'B}(\phi) = c_1 e^{\frac{3}{2}\phi} + \dots \qquad V_{SO(16)\times SO(16)}(\phi) = c_2\, e^{\frac{5}{2}\phi} + \dots$$

Main problem

[Dudas, Mourad, 2000]

The presence of the potential shifts the vacuum from flat space to a curved spacetime.

What can we say about the new vacuum of the theory?

Simplifying assumptions:

- We do not consider fluxes of any type,
- Since flat space is not a solution of the EOM anymore, we need to consider other spacetimes which are less symmetric,
- A common ansatz for the metric is then

$$ds^2 = e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2B(r)}dr^2 \ , \qquad \phi = \phi(r) \ .$$

Normalization

In the following it will be much easier to use the normalization

$$A = \frac{\mathcal{A}}{9} \qquad \phi = \frac{4\varphi}{3}$$

$$B = \mathcal{B} \qquad V(\phi) = \frac{16\,\mathcal{V}(\varphi)}{9}$$

With this choice of fields we have that

$$\mathcal{V}_{USp(32)}(\varphi) \sim e^{2\varphi} + \dots$$

$$\mathcal{V}_{0'B}(\varphi) \sim e^{2\varphi} + \dots$$

$$\mathcal{V}_{SO(16)\times SO(16)}(\varphi) \sim e^{\frac{10\varphi}{3}} + \dots$$

$$\mathcal{V}(\varphi) = \sum_{n\in\mathbb{N}} c_n e^{\left(\frac{4n}{3}+\frac{2}{3}\right)\varphi}$$

Equations of motion

Under these assumptions the system is described by a scalar-gravity lagrangian, with a potential for the dilaton field.

The equations of motion that determine the vacuum are then

$$\ddot{\mathcal{A}} - \dot{\mathcal{A}}\dot{\mathcal{B}} + \dot{\varphi}^2 = 0$$
$$\ddot{\varphi} + (\dot{\mathcal{A}} - \dot{\mathcal{B}})\dot{\varphi} - e^{2\mathcal{B}}\mathcal{V}'(\varphi) = 0$$
$$\dot{\varphi}^2 - 2e^{2\mathcal{B}}\mathcal{V}(\varphi) = \dot{\mathcal{A}}^2$$

For a generic potential these set of coupled ODEs cannot be integrated. However, to leading perturbative order in the theories of interest it can.

Dudas & Mourad vacuum

[Dudas, Mourad, 2000]

The solutions after this truncation are

- For the USp(32) and 0'B

$$e^{\varphi} = \alpha\,\sqrt{r}\,e^{c_1 r^2/2} \qquad\qquad e^{\mathcal{B}} = \frac{1}{\alpha\,\sqrt{r}}\,e^{-c_1 r^2/2}$$

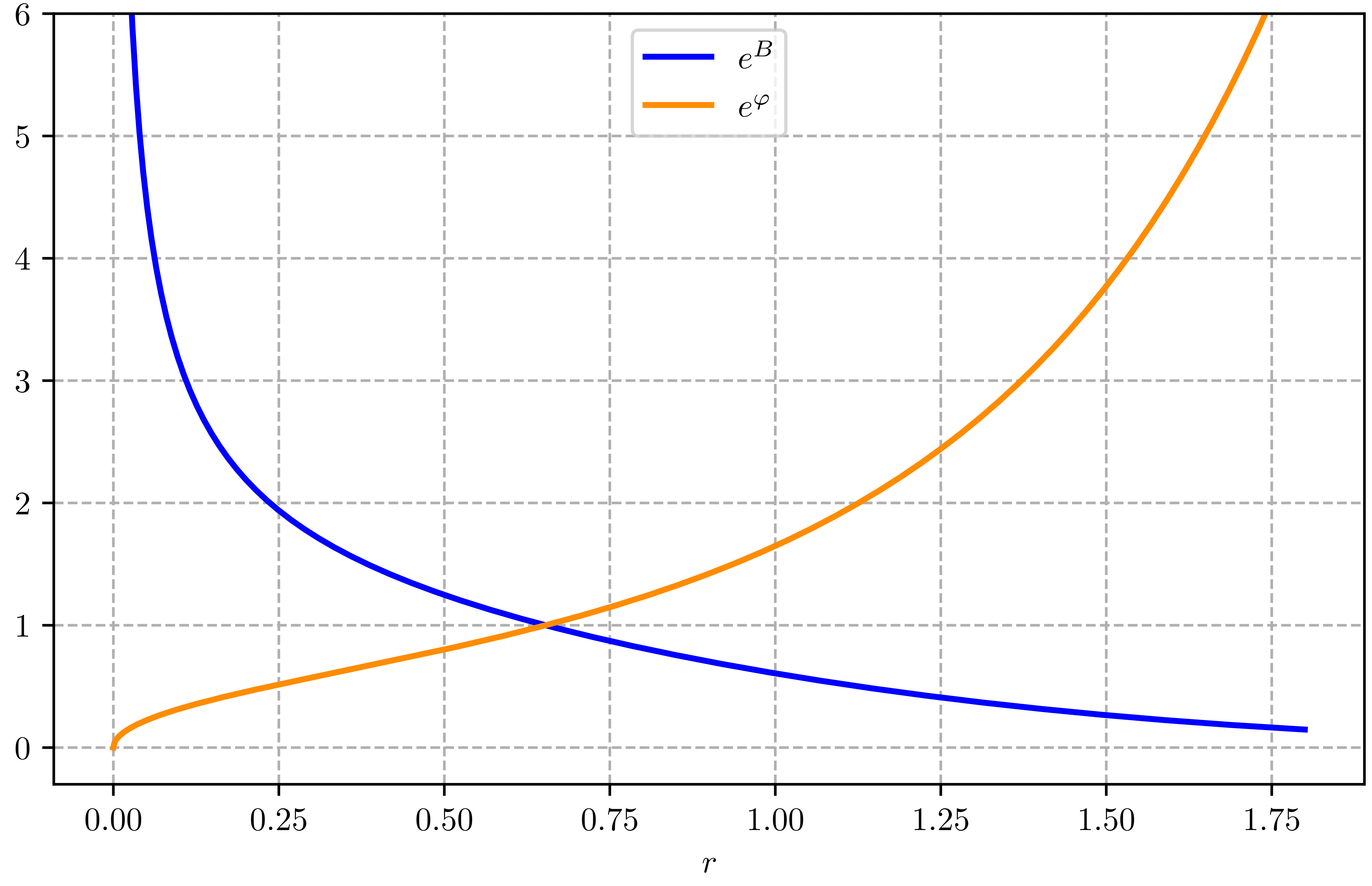
- For the SO(16)×SO(16)

$$e^{\varphi} = \alpha \sinh^{\frac{8}{3}}(\sqrt{c_2}r) \cosh^{\frac{3}{2}}(\sqrt{c_2}r) \qquad e^{\mathcal{B}} = \beta \sinh^{-\frac{5}{6}}(\sqrt{c_2}r) \cosh^{-\frac{15}{3}}(\sqrt{c_2}r)$$

This solution displays a dynamically compactified direction!

Dudas & Mourad vacuum

[Dudas, Mourad, 2000]



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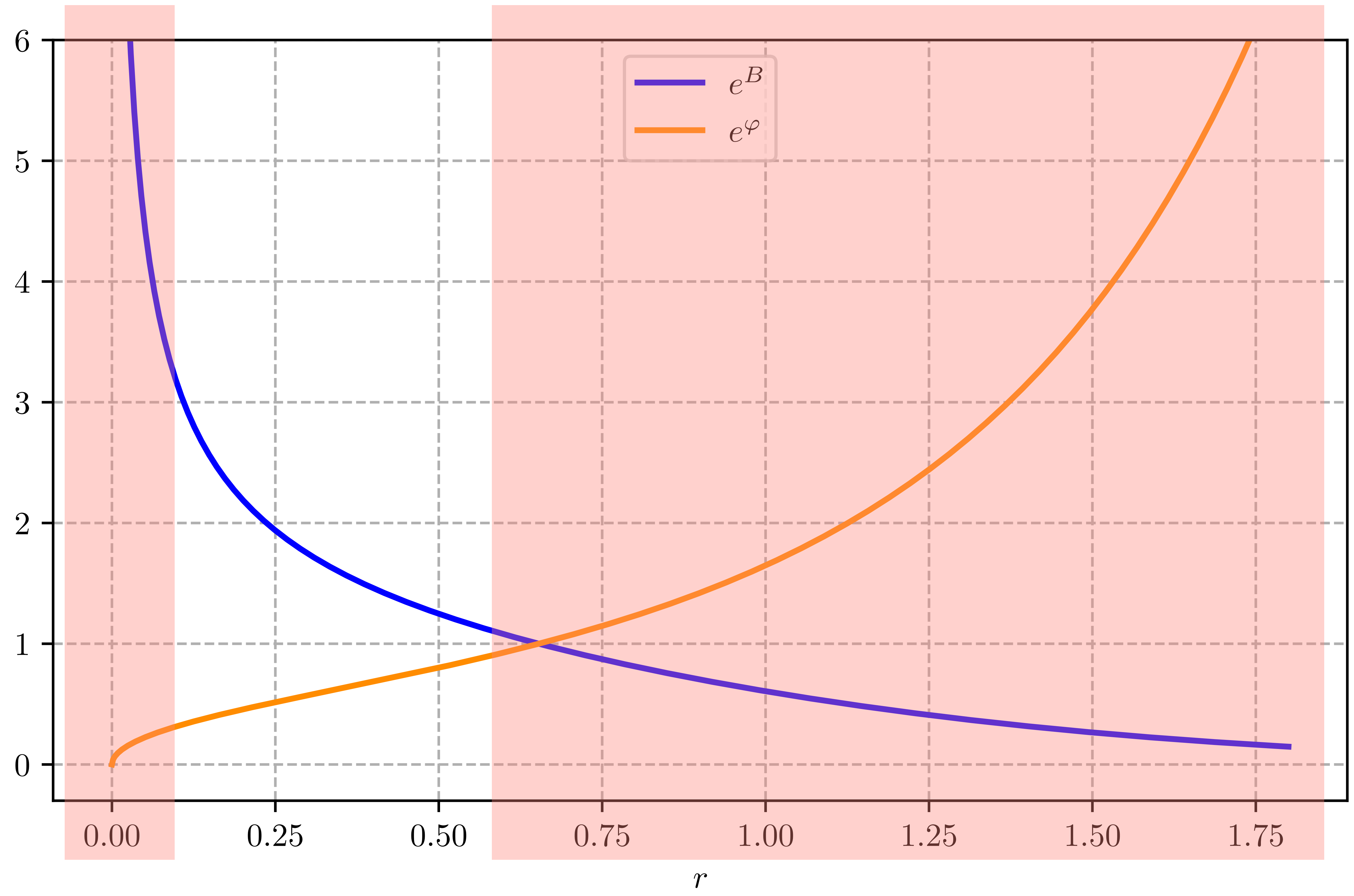


Motivating question: Considering higher genus contributions in the potential can we find a bounded string coupling?

Beyond Tree Level

Higher orders

The main idea is then to consider also the other perturbative contributions in the potential for the dilaton, thus

$$V(\phi) = \sum_{n\in\mathbb{N}} c_n e^{(n+\frac{1}{2})\phi}$$

However

- The coefficients are not known, and we don't even know how to compute them.
- The dynamical system is in general not integrable.

Therefore we would like to understand general rules that may grant a bounded string coupling, and then, to make the analysis simpler, we consider *string-like* potentials such that the dynamical system is integrable.

Hamiltonian constraint

[PP, Sagnotti, 2021]

One can find a general rule about potentials that can grant a bounded string coupling.

$$\frac{1}{2}\dot{\varphi}^2 - e^{2\mathcal{B}}\mathcal{V}(\varphi) = \frac{1}{2}\dot{\mathcal{A}}^2$$

This is reminiscent of the energy conservation law for one dimensional systems in classical mechanics, where the energy is positive.

This implies that

- If the potential is always positive then the system always reaches strong coupling,
- If the potential is negative in one region then it might happen that the string coupling is bounded above.

Climbing scalar

[Russo, 2004; Various works from Dudas, Kitazawa, Patil, Sagnotti, 2011-14]

This bounce back can be studied precisely, with a potential of the form

$$\mathcal{V}(\varphi) = -|\mathcal{V}_0|e^{2\gamma\varphi}$$

Solving the equations of motion one finds that

- For $\gamma \geq 1$ the dilaton always climbs the (inverted) potential and the string coupling is thus finite.
- Thus any negative higher genus correction in the potential points toward a boundedness of the string coupling.

The reason is also a matter of hierarchies.

Hierarchies

Consider our usual potential

$$V(\varphi) = \sum_{n \in \mathbb{N}} c_n e^{2\gamma_n \varphi}$$

If we want the potential to be defined for all possible values of the dilaton, the coefficients must decay fast enough.

Then writing $c_n = \mathrm{sign}(c_n) e^{-2\gamma_n \varphi_n}$ we get

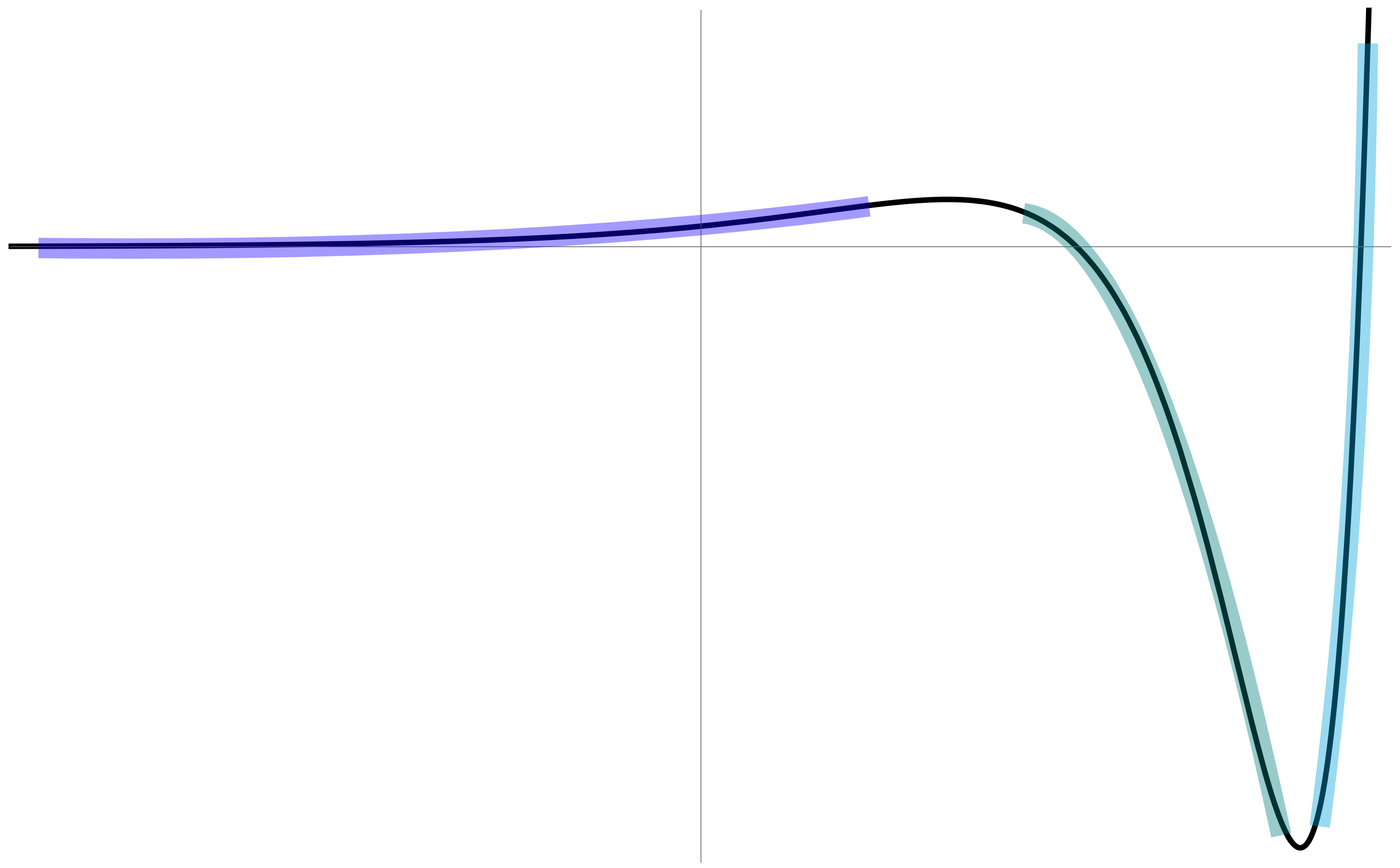
$$V(\varphi) = \sum_{n \in \mathbb{N}} \mathrm{sign}(c_n) e^{2\gamma_n (\varphi - \varphi_n)}$$

with $\varphi_{n+1} > \varphi_n$.

Hierarchies

This means that with negative coefficients we expect the potential to be of the form

$$V_\varphi = \dots + e^{2\gamma_{n-1}(\varphi-\varphi_{n-1})} - e^{2\gamma_n(\varphi-\varphi_n)} + e^{2\gamma_{n+1}(\varphi-\varphi_{n+1})} + \dots$$



The approximation of the climbing scalar can be trusted!

Thus in general a potential that has at least one coefficient of the expansion negative can bound the coupling

This can be analytically tested using integrable *string-like* potentials.

Potentials

[Fré, Sagnotti, Sorin, 2013]

$$V_1 = C\varphi e^{2\varphi}$$
$$V_2 = Ce^{2\varphi} + D$$
$$V_3 = Ce^{2\gamma\varphi} + De^{(\gamma+1)\varphi}$$
$$V_4 = C\left(e^{\frac{2}{\gamma}\varphi} - e^{2\gamma\varphi}\right)$$
$$V_5 = C\log(-\coth(\varphi)) + D$$
$$V_6 = C\cosh(\varphi) + D$$
$$V_7 = C\cosh^4\left(\frac{\varphi}{3}\right) + D\sinh^4\left(\frac{\varphi}{3}\right)$$
$$V_8 = \mathrm{Im}\left[C\log\left(\frac{e^{-2\varphi}+i}{e^{-2\varphi}-i}\right)\right] + D$$
$$V_9 = 2C\arctan(e^{2\varphi}) + D$$

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Elementary systems

Integrable setting $\mathcal{B} = 0$ and defining

$$\mathcal{A} = \log(xy) \qquad \varphi = \log\left(\frac{x}{y}\right)$$

one obtains

$$\ddot{x} = -\frac{Dx + Cy}{2}$$
$$\ddot{y} = -\frac{Dy + Cx}{2}$$
$$-2\dot{x}\dot{y} = Dxy + C\frac{x^2+y^2}{2}$$

Potentials

[Fré, Sagnotti, Sorin, 2013]

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Triangular systems

Example: for V_2 setting $\mathcal{B} = -\varphi$ and

$$\mathcal{A} = \frac{1}{2}\log(x) + v \qquad \varphi = \frac{1}{2}\log(x) - v$$

one obtains

$$\ddot{v} = -C$$
$$\ddot{x} = -2De^{2v}$$
$$\dot{x}\dot{v} = -De^{2v} - Cx$$

Potentials

[Fré, Sagnotti, Sorin, 2013]

$$V_1 = C\varphi e^{2\varphi}$$
$$V_2 = Ce^{2\varphi} + D$$
$$V_3 = Ce^{2\gamma\varphi} + De^{(\gamma+1)\varphi}$$
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Systems integrable via quadratures

Example: for V_5 setting $\mathcal{B} = -\mathcal{A}$ and

$$\mathcal{A} = \frac{1}{2}\log\left(\frac{\xi^2-\eta^2}{4}\right) \qquad \varphi = \frac{1}{2}\log\left(\frac{\xi-\eta}{\xi+\eta}\right)$$

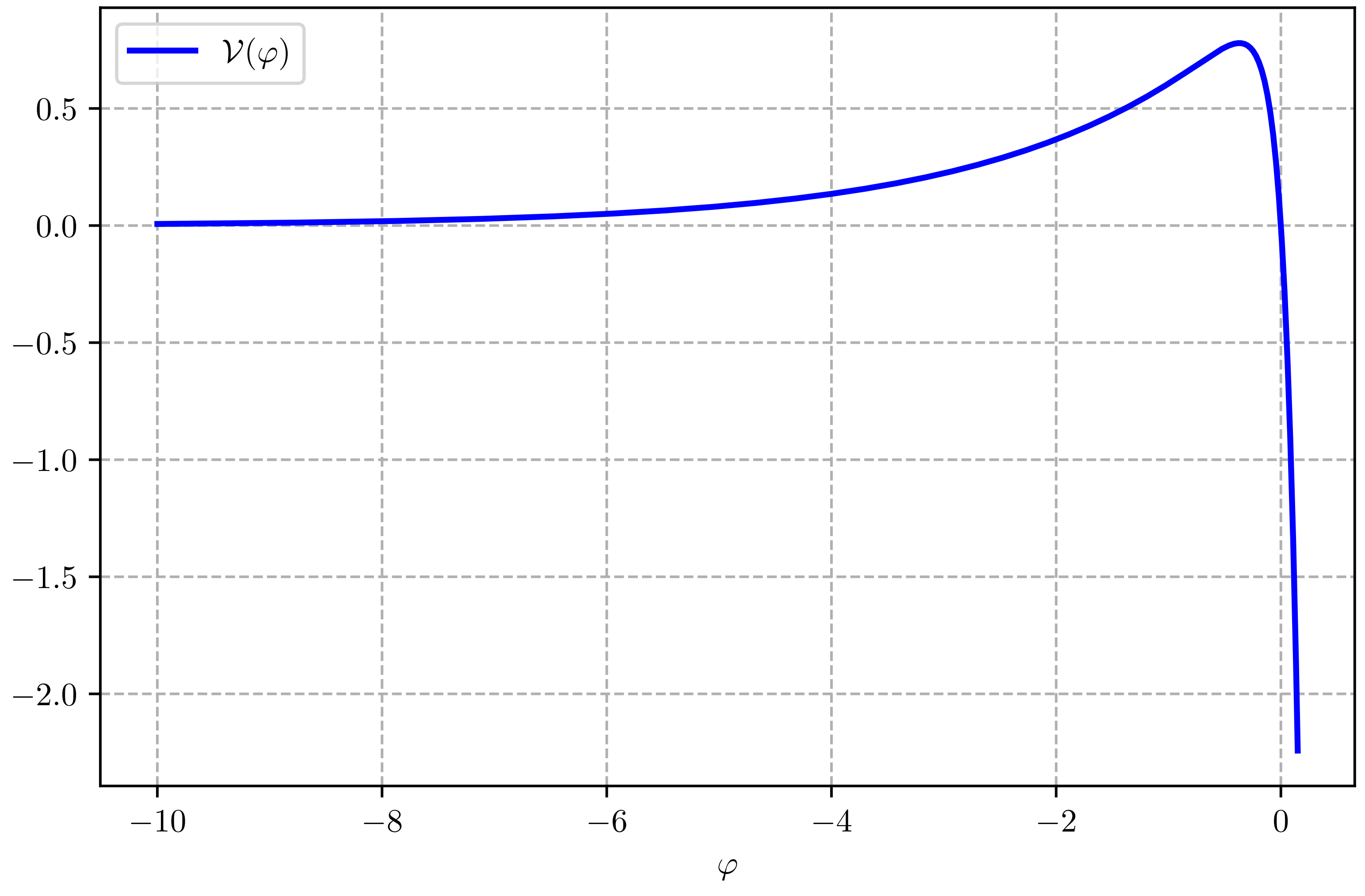
one obtains

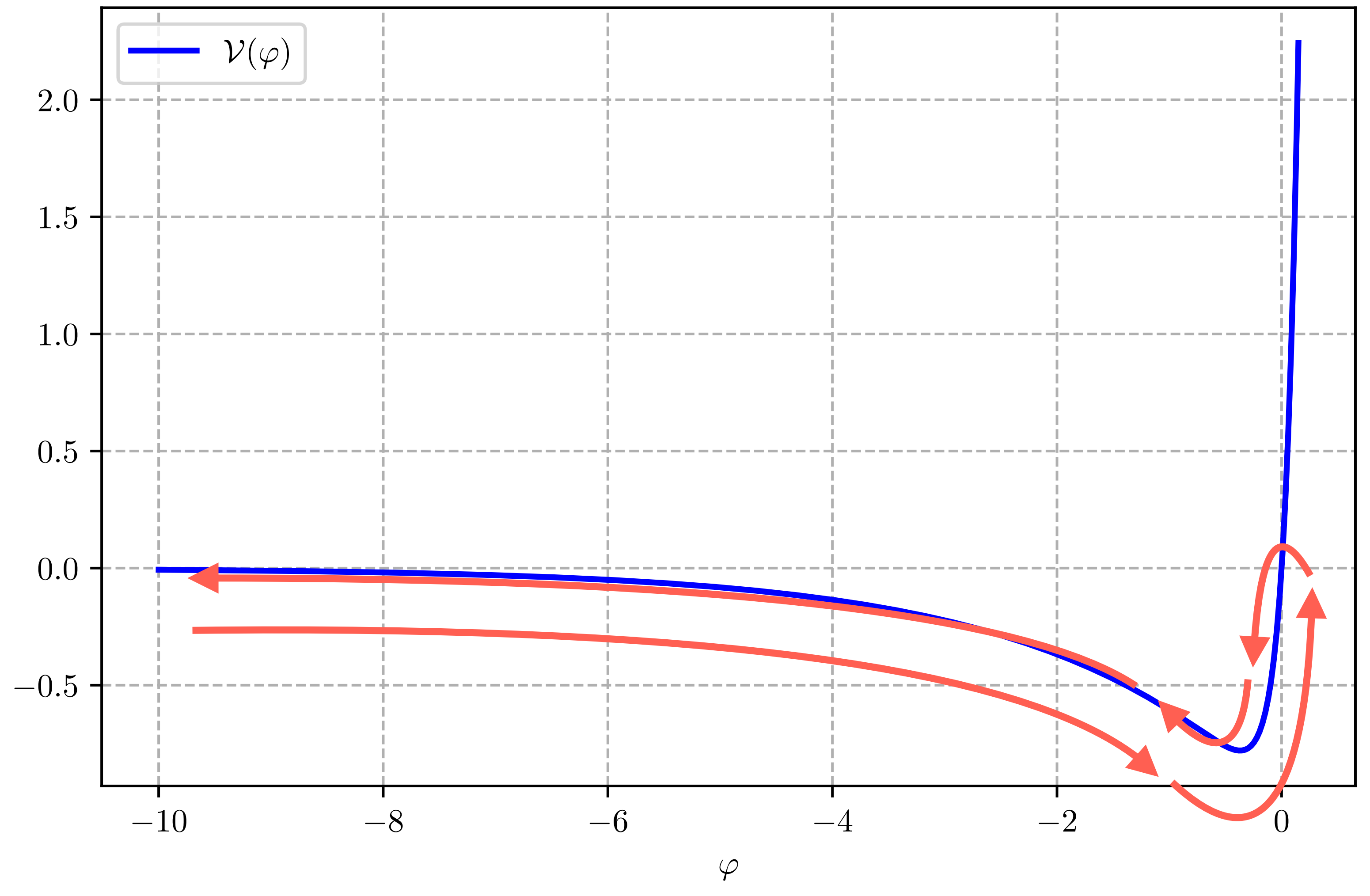
$$\ddot{\xi} = -\frac{4C}{\xi} \qquad \ddot{\eta} = -\frac{4C}{\eta}$$
$$\dot{\eta}^2 - \dot{\xi}^2 = 8C\log\left(\frac{\xi}{\eta}\right) + 8D$$

An example

Consider the potential

$$\mathcal{V}(\varphi) = C\left(e^{\frac{2}{\gamma}\varphi} - e^{2\gamma\varphi}\right)$$





$C = 1$

$\gamma = 4$

An example

Under the change of variables

$$\mathcal{B} = \mathcal{A} \,, \qquad \hat{\mathcal{A}} = \frac{1}{\sqrt{1-\gamma^2}}(\mathcal{A} + \gamma\varphi) \,, \qquad \hat{\varphi} = \frac{1}{\sqrt{1-\gamma^2}}(\varphi + \gamma\mathcal{A}) \,,$$

the equations of motion become

$$\ddot{\hat{\mathcal{A}}} = 2C\sqrt{1-\gamma^2}e^{2\sqrt{1-\gamma^2}\hat{\mathcal{A}}}$$

$$\ddot{\hat{\varphi}} = \frac{2C}{\gamma}\sqrt{1-\gamma^2}e^{\frac{2}{\gamma}\sqrt{1-\gamma^2}\hat{\varphi}}$$

$$\dot{\hat{\mathcal{A}}}^2 - \dot{\hat{\varphi}}^2 = 2Ce^{2\sqrt{1-\gamma^2}\hat{\mathcal{A}}} - 2Ce^{\frac{2}{\gamma}\sqrt{1-\gamma^2}\hat{\varphi}}$$

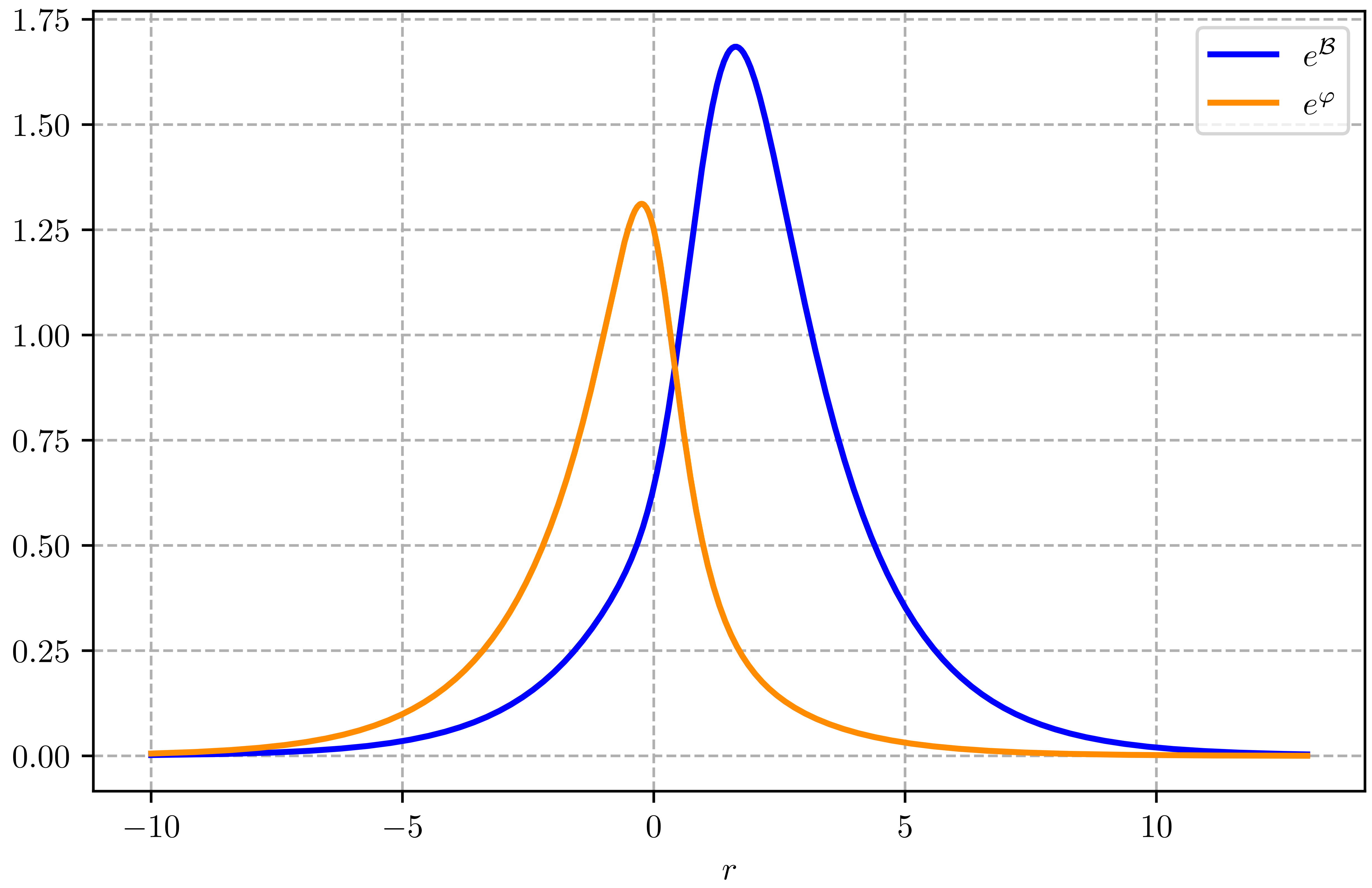
An example

[PP, Sagnotti, 2021]

The solution is

$$e^{\mathcal{A}} = e^{\mathcal{A}_0} \frac{\left[\cosh(\omega(r - r_{\hat{\varphi}}))\right]^{\frac{\gamma^2}{1-\gamma^2}}}{\left[\cosh(\gamma\omega(r - r_{\hat{\mathcal{A}}}))\right]^{\frac{1}{1-\gamma^2}}} = e^{\mathcal{B}}$$

$$e^{\varphi} = e^{\varphi_0} \frac{\left[\cosh(\gamma\omega(r - r_{\hat{\mathcal{A}}}))\right]^{\frac{\gamma}{1-\gamma^2}}}{\left[\cosh(\omega(r - r_{\hat{\varphi}}))\right]^{\frac{\gamma}{1-\gamma^2}}}$$



Solutions are also perturbatively stable!

Positive coefficients

[PP, Sagnotti, 2021]

Question: what happens if all the coefficients are positive?

- If the potential is defined for all values of φ then the system is guaranteed to attain values of strong coupling, because of the Hamiltonian constraint.
- However, if the coefficients of the expansion behave at least geometrically, then the potential stops for some φ^* and the coupling is bounded above!

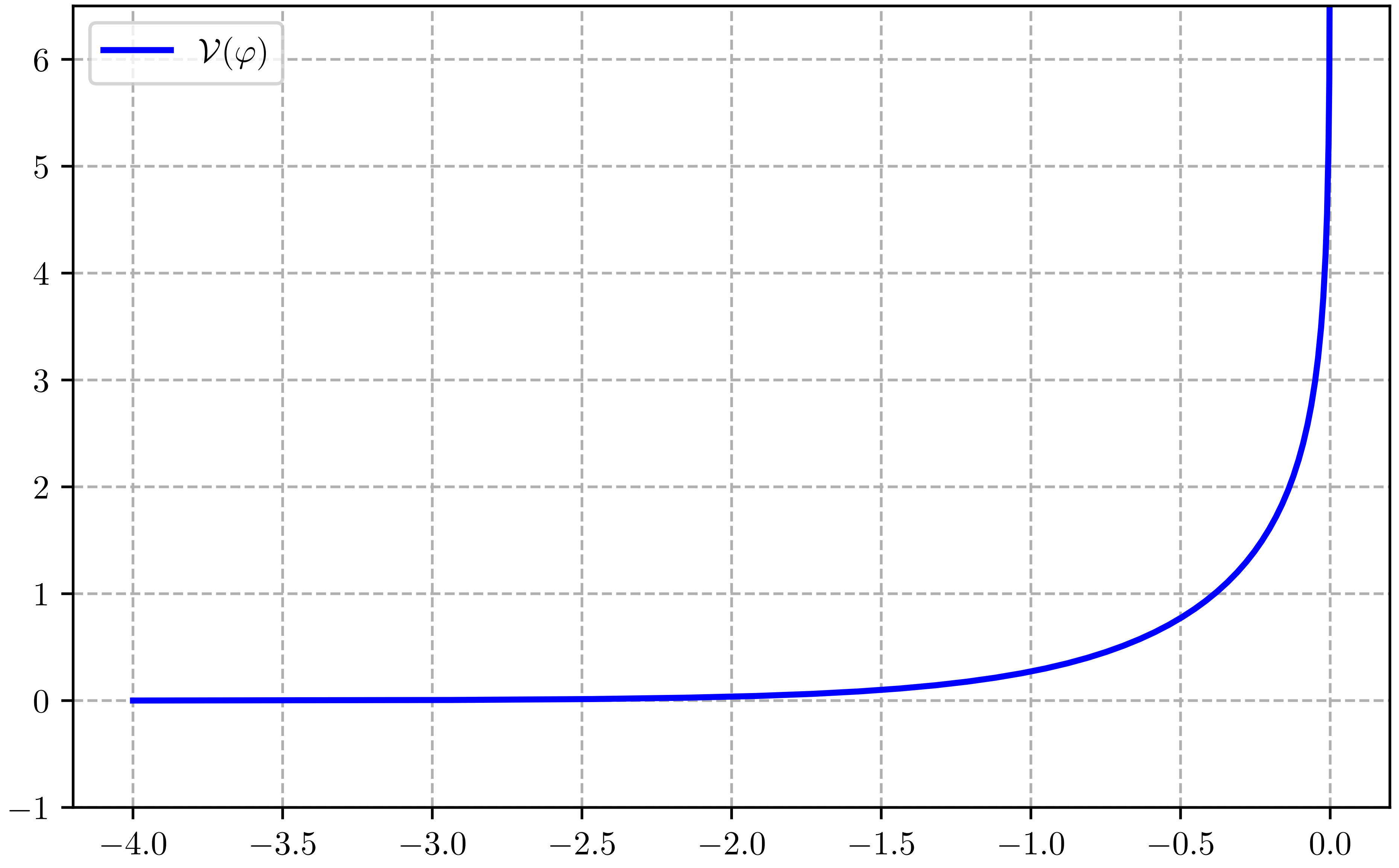
Then the natural question is whether one can still have a spontaneous compactification.

A second example

Consider the potential

$$\mathcal{V}(\varphi) = C \log(-\coth(\varphi)) + D$$

This potential does not have only positive coefficients, but we can argue that the solution should be similar.



A second example

Under the change of variables

$$\mathcal{B} = -\mathcal{A} \,, \qquad \mathcal{A} = \frac{1}{2}\log\left(\frac{\xi^2 - \eta^2}{4}\right) \,, \qquad \varphi = \frac{1}{2}\log\left(\frac{\xi - \eta}{\xi + \eta}\right) \,,$$

the equations of motion become

$$\ddot{\xi} = -\frac{4C}{\xi} \,, \qquad \ddot{\eta} = -\frac{4C}{\eta} \,,$$

$$\dot{\eta}^2 - \dot{\xi}^2 = 8C\log\left(\frac{\xi}{\eta}\right) + 8D$$

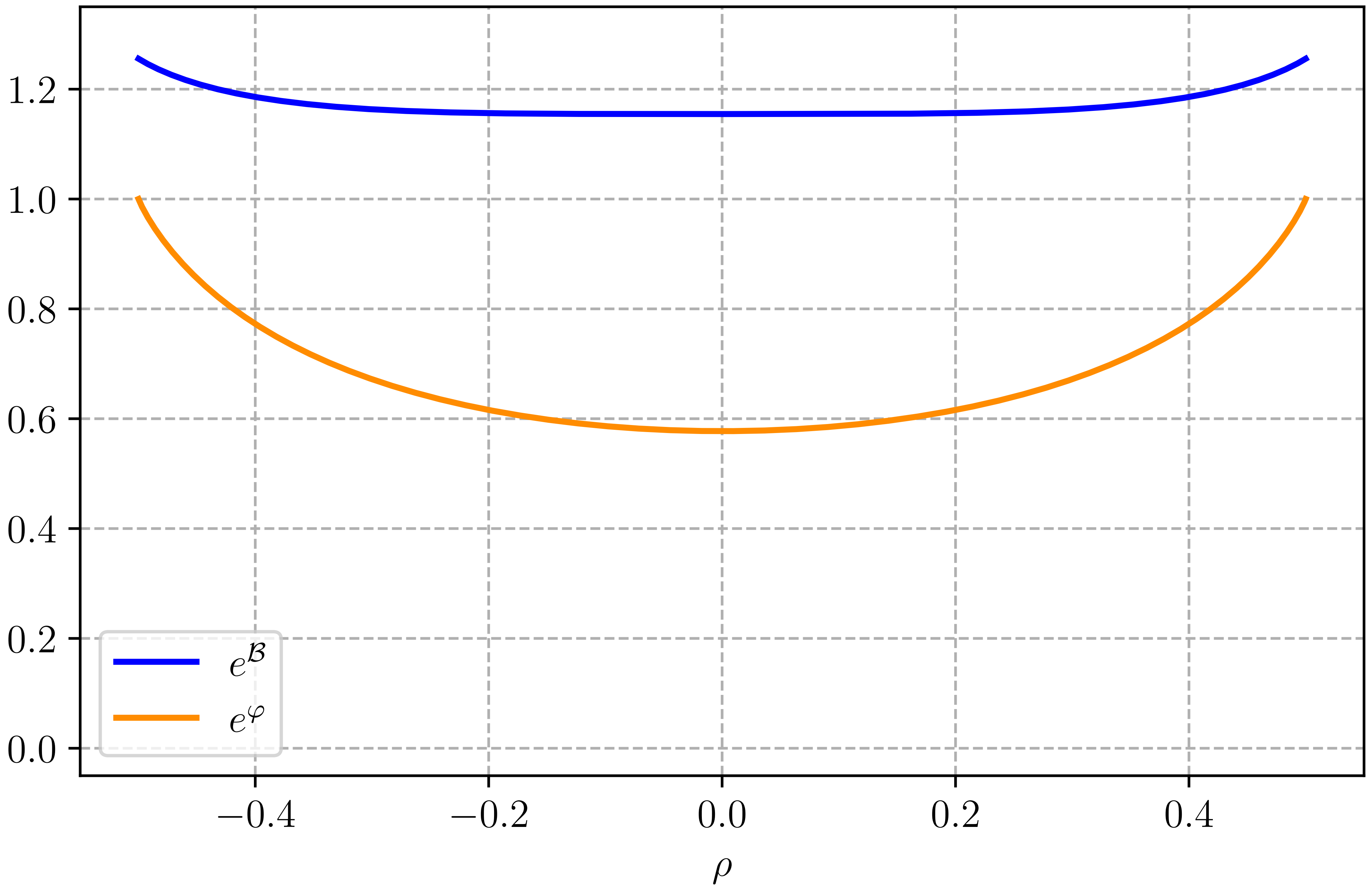
A second example

[PP, Sagnotti, 2021]

The solution is

$$e^{\mathcal{A}} = \frac{\xi_0}{2}\sqrt{1 - f^2 \exp\left\{2\left[\text{erf}^{-1}(\rho-\alpha)\right]^2 - 2\left[\text{erf}^{-1}\left(\frac{\rho}{f}\right)\right]^2\right\}}\exp\left\{-\left[\text{erf}^{-1}(\rho-\alpha)\right]^2\right\} ,$$

$$e^{\varphi} = \sqrt{\frac{1 - f\exp\left\{\left[\text{erf}^{-1}(\rho-\alpha)\right]^2 - \left[\text{erf}^{-1}\left(\frac{\rho}{f}\right)\right]^2\right\}}{1 + f\exp\left\{\left[\text{erf}^{-1}(\rho-\alpha)\right]^2 - \left[\text{erf}^{-1}\left(\frac{\rho}{f}\right)\right]^2\right\}}} , \qquad \mathcal{B} = -\mathcal{A} .$$



Concluding Remarks

Lessons for String Theory

- Non-supersymmetric string theories display a multitude of interesting phenomena.
- One of the most interesting features is a (supposedly) dynamical compactification, which goes in the right phenomenological direction.
- Our study points toward the fact that considering higher genus contributions in the expansion of the potential it is generally not difficult to have a perturbative string coupling for these models, and thus solutions that can be studied precisely.
- Remaining open questions are

  1. Considering also higher derivative terms in the effective action, can we have also small curvatures?
  2. Can we find other spontaneous compactifications? Do these results also apply there?

Thank you for you attention!

Spectrum of theories

$USp(32)$	$U(32)$ 0′B	$SO(16) \times SO(16)$
$g_{\mu\nu}$, φ , $B'_{\mu\nu}$	$g_{\mu\nu}$, φ , $B'_{\mu\nu}$	$g_{\mu\nu}$, φ , $B_{\mu\nu}$
A^a_μ	A^a_μ	A^a_μ
λ_L in $\mathbf{496}$	λ_L in $\mathbf{496}$	λ_L in $(\mathbf{128},\mathbf{1})+(\mathbf{1},\mathbf{128})$
ψ^μ_R	$A^+_{\mu\nu\rho\sigma}$, a	λ_R in $(\mathbf{16},\mathbf{16})$