Islands in AdS/ICFT

Based on ArXiv: 2202.11718 in collaboration with T. Anous, M. Meineri and J. Sonner

UNIVERSITÉ DE GENÈVE

SwissMAP
The Mathematics of Physics
National Centre of Competence in Research

Pietro Pelliconi - SwissMAP General Meeting, Les Diablerets, 14/09/2022

Holographic duality

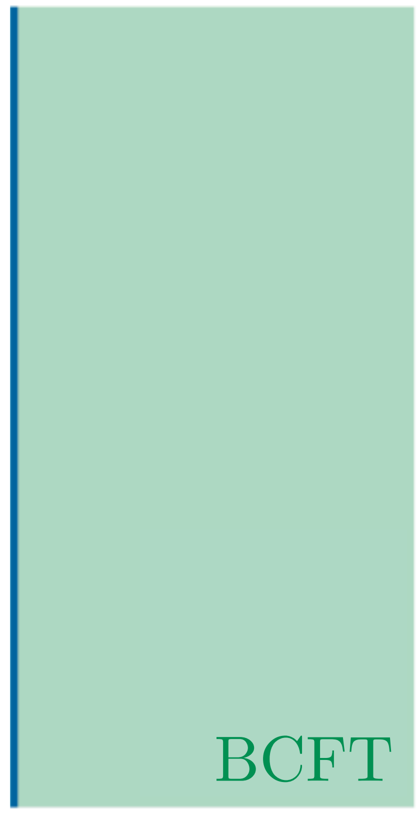
- AdS/CFT gives a non-perturbative definition of Quantum Gravity in Anti de-Sitter.
- It relates Quantum Gravity in AdS with a Conformal Field Theory in one dimension lower.
- A weakly coupled gravitational theory is obtained as a "large-N" limit. For example, in AdS_3/CFT_2

$$c = \frac{3}{2} \frac{L}{G_{(3)}}$$

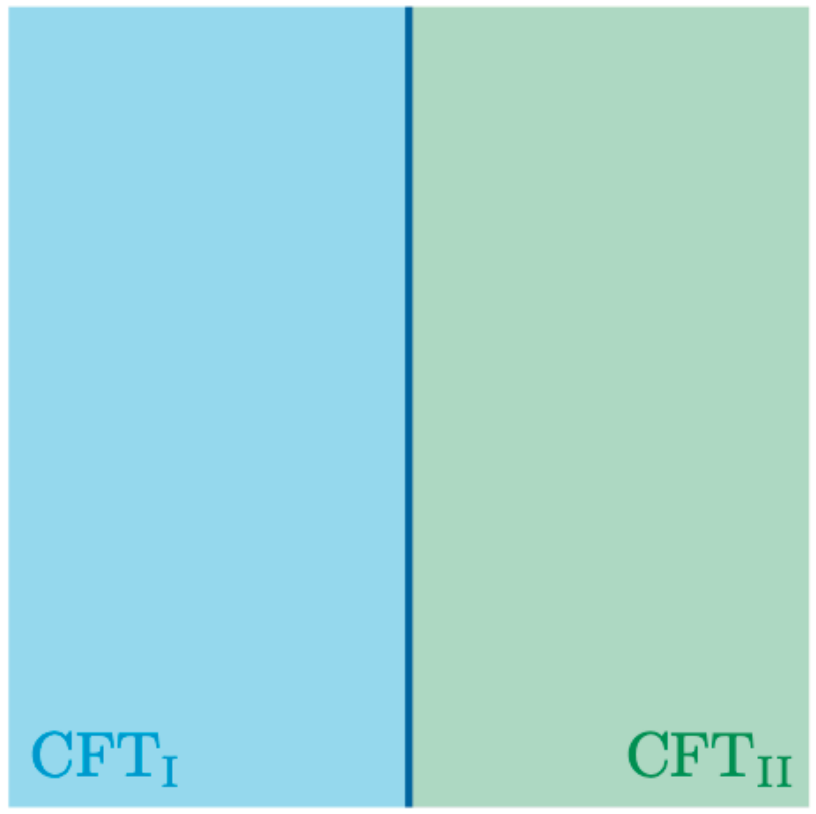
[Maldacena 1998; Brown, Henneaux, 1986]

BCFTs and ICFTs

- Interesting setups are Boundary and Interface CFTs



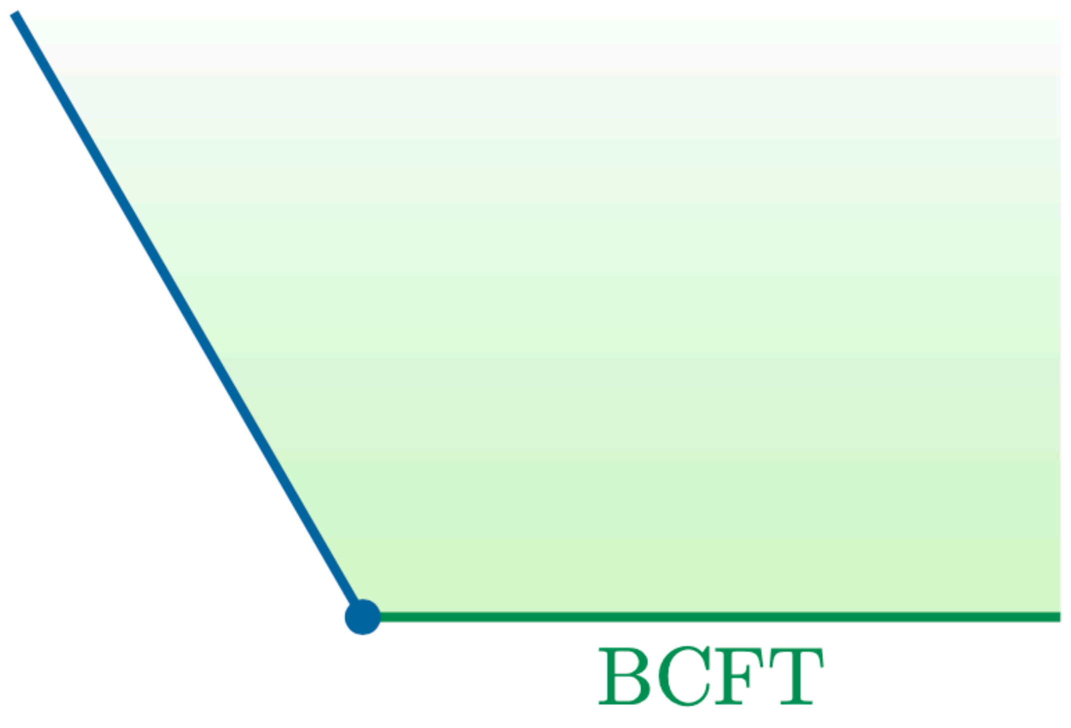
Boundary CFT



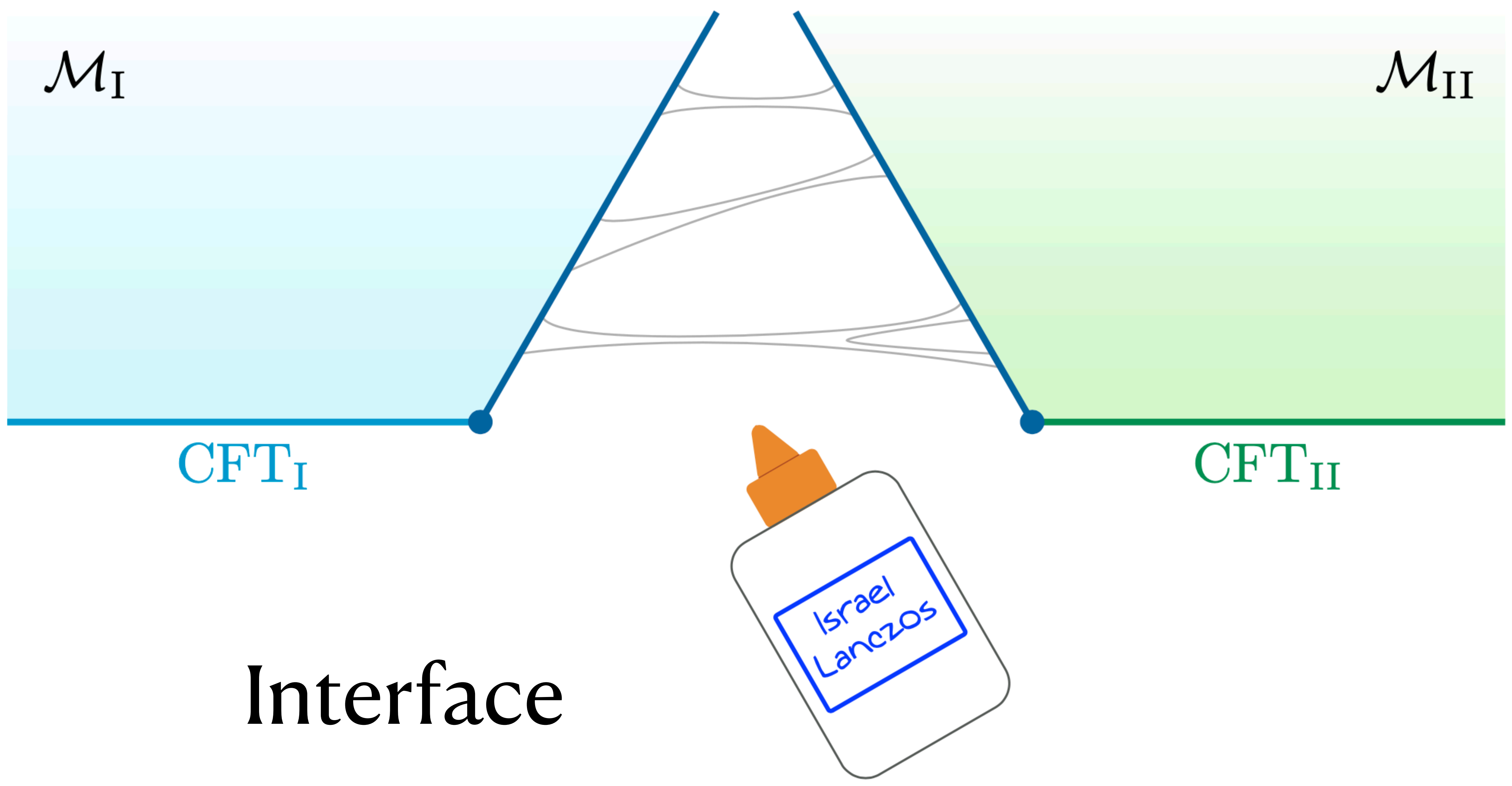
Interface CFT

KRS Braneworlds

- Some of their holographic dual are supposedly Karch-Randall-Sundrum braneworlds



Boundary



Interface

[Karch, Randall, 2000-01; Takayanagi, 2011; Bachas, de Boer, Dijkgraaf, Ooguri, 2001]

KRS Braneworlds

- We can parametrize

$$\mathrm{d}s^2_{\mathrm{AdS}_{(3)}} = \mathrm{d}\rho^2 + \mathrm{d}s^2_{\mathrm{AdS}_{(2)}}$$

- This means that on the brane

$$\frac{1}{G_{(2)}} = \frac{\rho^*}{G_{(3)}}$$

- Weakly coupled gravity on the brane is then the limit

$$\rho^* \to \infty$$

Braneworlds and Holography

- We then have two expansion parameters, and physical observables will be functions of these

$$\mathcal{O} = \mathcal{O}(c, \rho^*)$$

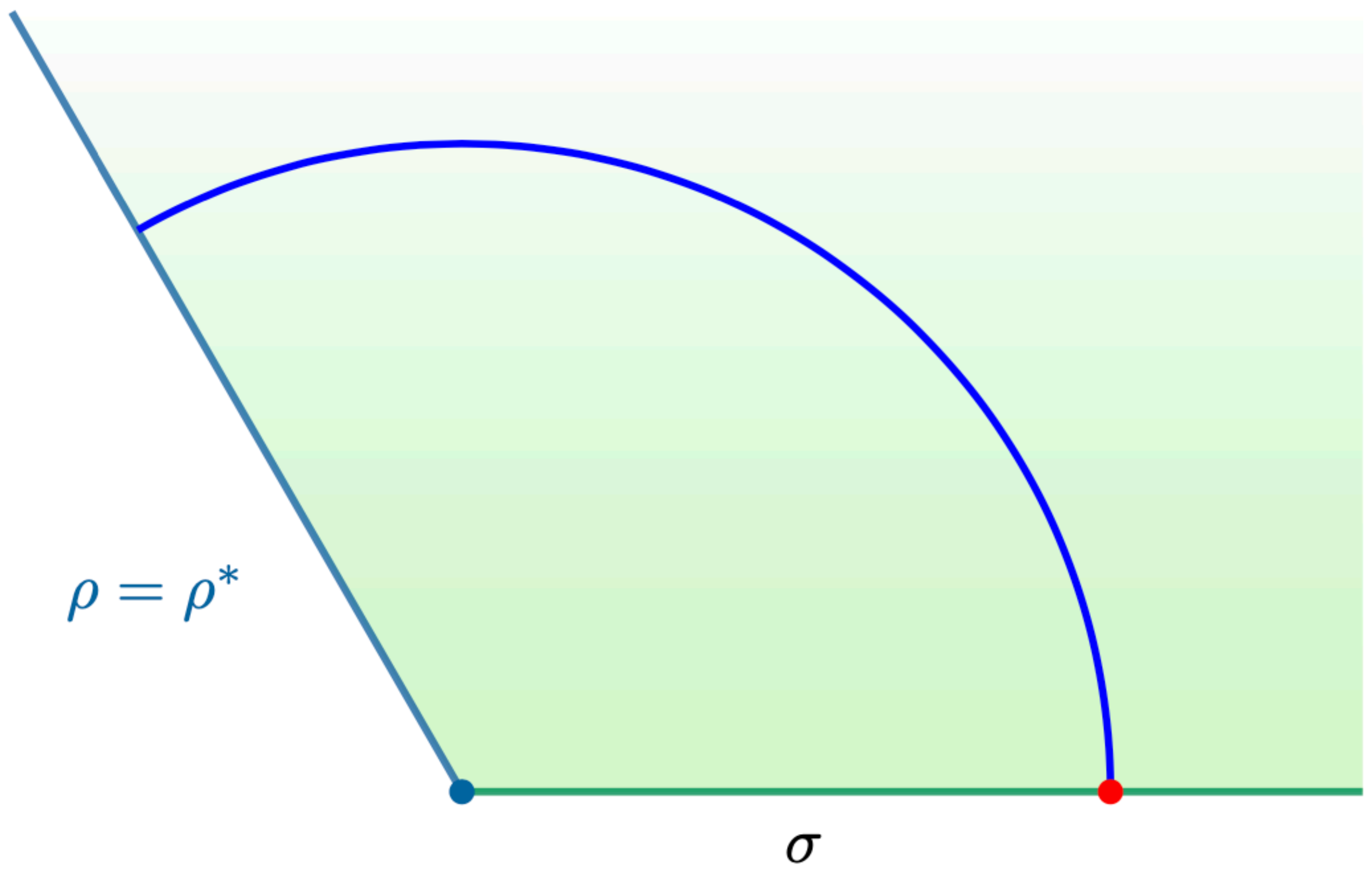
or alternatively

$$\mathcal{O} = \mathcal{O}(G_{(3)}, G_{(2)})$$

- Through the 3D holographic duality we can compute the large-c leading term, and obtain the leading term in $G_{(2)}$

Entanglement entropy in AdS/BCFT

- To give an example, let's compute the entanglement entropy in AdS/BCFT
- We can use the RT prescription to compute the answer at large c



$$S = \frac{c}{6}\log\left(\frac{2\sigma}{\varepsilon}\right) + \frac{c\,\rho^*}{6L}$$

or

$$S = \frac{L}{4G_{(3)}}\log\left(\frac{2\sigma}{\varepsilon}\right) + \frac{1}{4G_{(2)}}$$

[Takayanagi, 2011]

Islands in AdS/BCFT

- How do we interpret this result from the point of view of the braneworld?
- In the limit of large ρ^* the brane approaches the boundary and the dynamics becomes conformal. Therefore, if $\mathrm{d}s^2 = \Omega(y,\tau)^{-2}(\mathrm{d}y^2 + \mathrm{d}\tau^2)$

$$S = \frac{c}{6}\log\left(\frac{|y_1 - y_2|^2}{\Omega(y_1)\Omega(y_2)\varepsilon_1\varepsilon_2}\right)$$

- Gravity seems to tell us that we should extremize over the point on the brane

$$S = \mathrm{ext}_y \frac{c}{6}\log\left(\frac{(\sigma+y)^2}{y\varepsilon}\cosh(\rho^*/L)\right)$$

[Penington, 2019; Almheiri, Engelhardt, Marolf, Maxfield, 2019; Almheiri, Mahajan, Maldacena, Zhao, 2019; Suzuki, Takayanagi, 2022; Anous, Meineri, PP, Sonner, 2022]

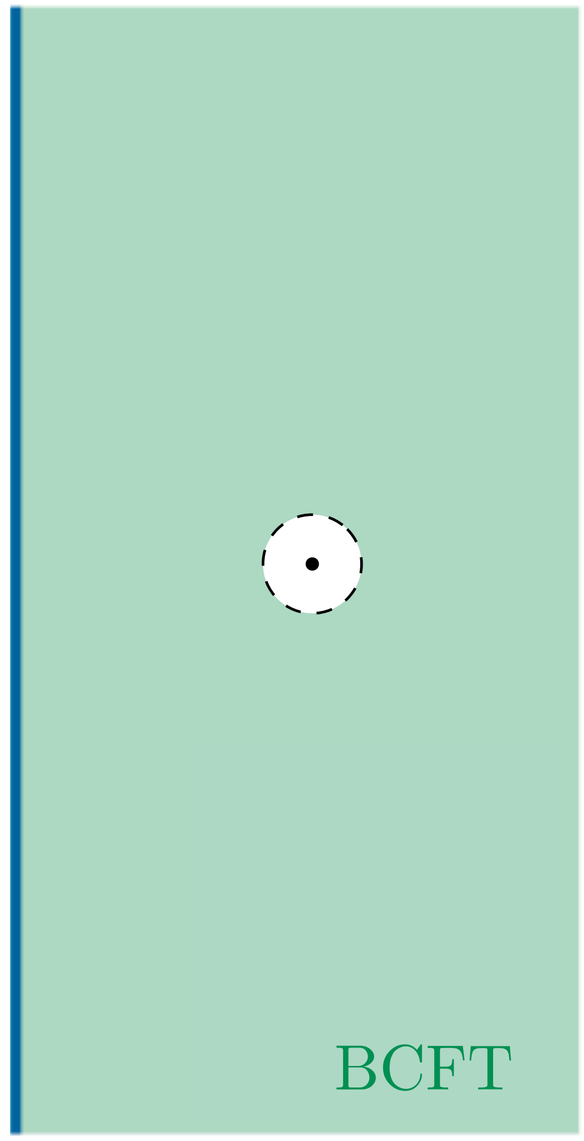
Islands in AdS/BCFT

- The extremization gives

$$S = \frac{c}{6}\log\left(\frac{2\sigma}{\varepsilon}\right) + \frac{c\,\rho^*}{6L} + \dots$$

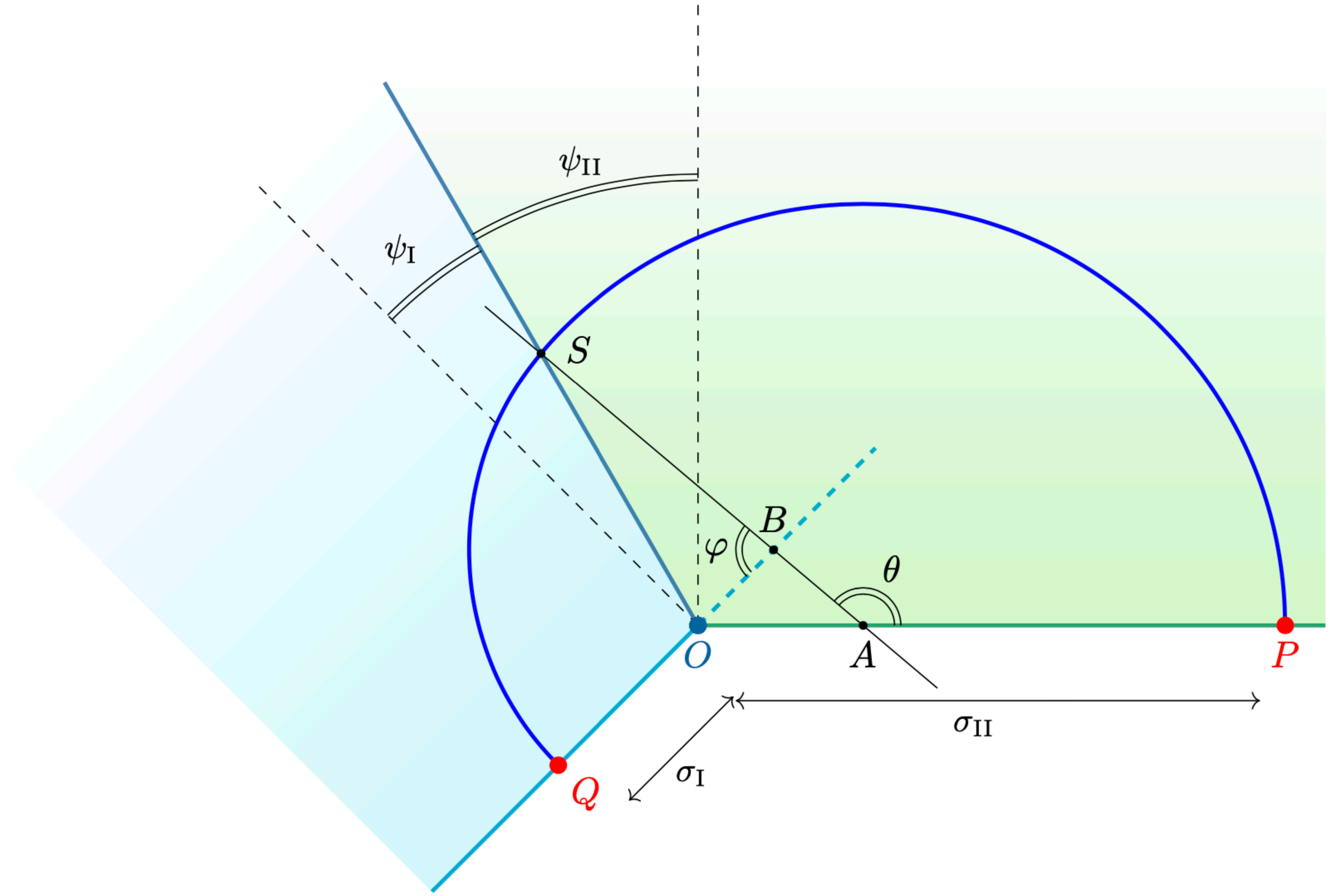
which, in the limit of large ρ^* agrees with the 3D computation.

- This match could have been anticipated, since we can compute this observable using a one-point function of a twist operator, which in BCFTs is completely determined by the symmetries of the system.



Entanglement entropy in AdS/ICFT

- What is not determined by the symmetries are entanglement entropies in AdS/ICFT



$$S = \frac{c_{\rm I}}{6}\log\left[\frac{2r}{\varepsilon_{\rm I}}\tan\left(\frac{\varphi}{2}\right)\right] + \frac{c_{\rm II}}{6}\log\left[\frac{2R}{\varepsilon_{\rm II}}\tan\left(\frac{\theta}{2}\right)\right]$$

[Bachas, Chapman, Ge, Policastro, 2020; Anous, Meineri, PP, Sonner, 2022]

Entanglement entropy in AdS/ICFT

$$S = \frac{c_{\text{I}}}{6} \log\left(\frac{2r}{\varepsilon_{\text{I}}} \tan\left(\frac{\phi}{2}\right)\right) + \frac{c_{\text{II}}}{6} \log\left(\frac{2R}{\varepsilon_{\text{II}}} \tan\left(\frac{\theta}{2}\right)\right)$$

$$\varphi = \pi + \psi_{\text{I}} + \psi_{\text{II}} - \theta$$

$$r = \frac{1}{2}\csc\left(\frac{\varphi}{2}\right)\sec\left(\frac{\psi_{\text{I}}+\psi_{\text{II}}}{2}\right)\left[\sigma_{\text{II}}\cos\left(\frac{\theta}{2}\right) - \sigma_{\text{I}}\cos\left(\frac{\theta}{2}+\varphi\right)\right] \qquad R = \frac{1}{2}\csc\left(\frac{\theta}{2}\right)\sec\left(\frac{\psi_{\text{I}}+\psi_{\text{II}}}{2}\right)\left[\sigma_{\text{I}}\cos\left(\frac{\varphi}{2}\right) - \sigma_{\text{II}}\cos\left(\frac{\varphi}{2}+\theta\right)\right]$$

$$\cos(\theta) = \frac{\cos\left(\frac{\psi_{\text{I}}-\psi_{\text{II}}}{2}\right)}{\sigma_{\text{I}}^2+\sigma_{\text{II}}^2+2\sigma_{\text{I}}\sigma_{\text{II}}\cos(\psi_{\text{I}}+\psi_{\text{II}})}\Bigg\{-\sigma_{\text{II}}^2\cos\left(\frac{\psi_{\text{I}}-\psi_{\text{II}}}{2}\right)+\sigma_{\text{I}}^2\cos\left(\frac{\psi_{\text{I}}+3\psi_{\text{II}}}{2}\right)+2\sigma_{\text{I}}\sigma_{\text{II}}\sin(\psi_{\text{II}})\sin\left(\frac{\psi_{\text{I}}+\psi_{\text{II}}}{2}\right)$$
$$-\left[\sigma_{\text{I}}\sin\left(\frac{\psi_{\text{I}}+3\psi_{\text{II}}}{2}\right)-\sigma_{\text{II}}\sin\left(\frac{\psi_{\text{I}}-\psi_{\text{II}}}{2}\right)\right]\sqrt{\left[\frac{(\sigma_{\text{I}}+\sigma_{\text{II}})^2-(\sigma_{\text{I}}-\sigma_{\text{II}})^2\cos(\psi_{\text{I}}-\psi_{\text{II}})+4\sigma_{\text{I}}\sigma_{\text{II}}\cos(\psi_{\text{I}}+\psi_{\text{II}})}{2\cos^2\left(\frac{\psi_{\text{I}}-\psi_{\text{II}}}{2}\right)}\right]}\Bigg\}$$

Islands in AdS/ICFT

- To interpret this result we can again use the island formula, which in this case reads

$$S = \text{ext}_y \left[\frac{c_{\rm I}}{6} \log \left(\frac{(y+\sigma_{\rm I})^2}{y\,\varepsilon} \frac{1}{\cos(\psi_{\rm I})} \right) + \frac{c_{\rm II}}{6} \log \left(\frac{(y+\sigma_{\rm II})^2}{y\,\varepsilon} \frac{1}{\cos(\psi_{\rm II})} \right) \right]$$

- The extremization gives

$$y^* = \frac{(c_{\rm I} - c_{\rm II})(\sigma_{\rm I} - \sigma_{\rm II}) + \sqrt{(c_{\rm I} - c_{\rm II})^2(\sigma_{\rm I} - \sigma_{\rm II})^2 + 4\sigma_{\rm I}\sigma_{\rm II}(c_{\rm II} + c_{\rm II})^2}}{2(c_{\rm I} + c_{\rm II})}$$

- In the limit where $\rho^*_{\rm I}$ and $\rho^*_{\rm II}$ are large, the entropy agrees with the previous result.

[Anous, Meineri, PP, Sonner, 2022]

Conclusions

- Braneworlds in holography are interesting scenarios in which we can compute observables in the induced gravitational coupling constant.
- This gives us a way to understand features of quantum gravity, which can teach lessons for frameworks where braneworlds are not present.
- For example, we have shown how the island formula gives the correct result also for observables which are not completely determined by the symmetries, which is a strong check of the prescription.

Thank you!